

Longitudinal Targets in Data Envelopment Analysis

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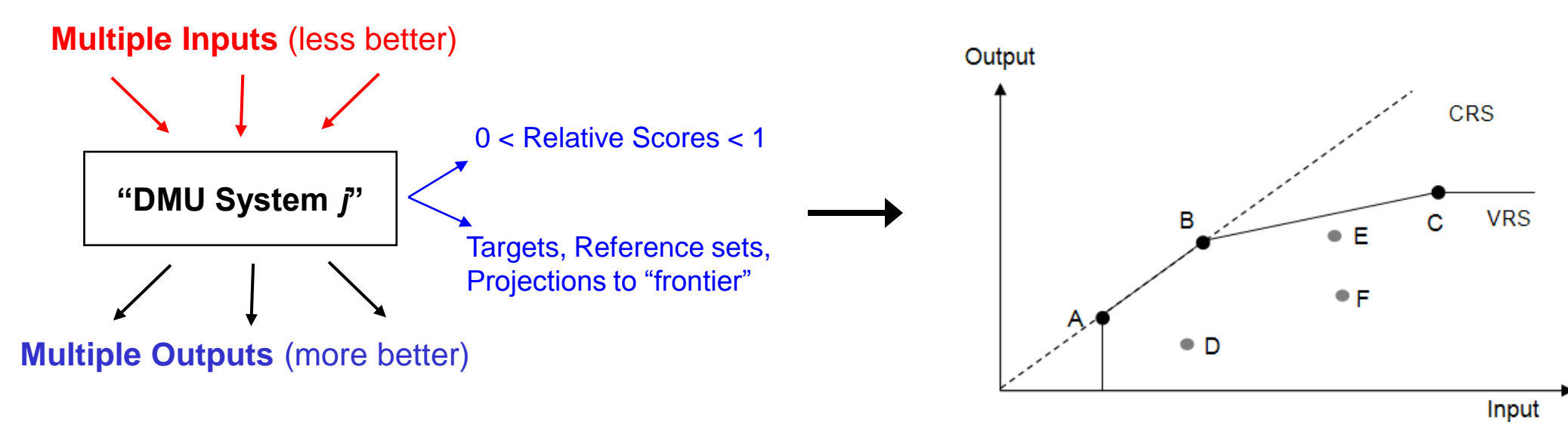
Objective

- Use data envelopment analysis (DEA) to evaluate and rank relative performance of drug stores and identify pathways to improvement .
- Set longitudinal targets for decision making units (DMUs) with considering uncertain future behaviors of other DMUs.

METHOD

Data Envelopment Analysis

DEA determines the relative efficiency of k multiple systems (drug stores) at consuming multiple inputs (energy, rent, admin, material, wages, and cost) to produce multiple outputs (profit, etc)



- Developed by Charnes , Cooper and Rhodes (CCR)
- Expanded by Banker , Charnes and Cooper (BCC)
- Many variations developed since then

Mathematical Formulation

In a DEA model where a set of n observations is considered on the DMUs, each observation uses m inputs to produce s outputs where v represents input weights and u represents output weights. Input-oriented, variable returns to scale models were used throughout analysis.

Envelopment Model

$$\max \sum_{i=1}^m v_i x_{io} + \mu$$

$$s.t. \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s u_r y_{rj} + v \geq 0$$

$$\sum_{r=1}^s u_r y_{ro} = 1$$

$$u_r, v_i \geq 0$$

$$\mu : \text{unrestricted}$$

Multiplier Model

$$\min \theta - \varepsilon (\sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+)$$

$$s.t. \sum_{j=1}^n \lambda_j x_{ij} + s_i^- = \theta x_{io} \quad i=1,2,\dots,m;$$

$$\sum_{j=1}^n \lambda_j x_{ij} + s_i^- = x_{io} \quad i \notin I;$$

$$\sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = y_{ro} \quad r=1,2,\dots,s;$$

$$\lambda_j \geq 0 \quad j=1,2,\dots,n.$$

$$\sum_{j=1}^n \lambda_j = 1$$

VRS-I Models

$$\text{Input Targets: } X_{ij} = \sum \lambda_k X_{ik} \quad i=1, 2, \dots, m$$

$$\text{Output Targets: } Y_{ro} = \sum \lambda_k Y_{rk} \quad j=1, 2, \dots, n$$

ILLUSTRATION OF PROBLEM

Input and output targets for each inefficient DMU that would move them onto the efficiency frontier *if and only if* other DMUs continue to produce the same levels of inputs and outputs.

Drugstore (DS) example by Parkan et al.

Monthly data (18) for 8 decision making units , Six inputs and one output

DS	1	14	Comments
D1	0.939	1	Inefficient → Efficient
D6	0.934	0.860	Inefficient → Inefficient
D2	1	1	Efficient → Inefficient
D5	1	0.820	Efficient → Efficient

Efficiency Scores Using First Month's Targets in all Subsequent Periods

RESULTS: MODEL EXTENSIONS

1. Periods over time approach

1.1 Next period over time

Use the next period data for given DMU and period 1 data for all others

DS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
D1	0.974	0.928	0.943	0.983	1	1	0.849	1	0.979	1	0.973	0.978	1	0.961	1	1	1
D2	1	1	1	1	1	1	1	1	0.958	1	1	1	1	1	1	1	1
D5	1	1	1	1	1	1	0.972	1	1	1	0.956	0.871	1	1	1	1	0.988
D6	0.897	0.918	0.777	0.880	0.807	0.776	0.750	0.723	0.762	0.857	0.806	0.894	0.949	0.903	0.924	0.860	0.854

1.2 Past period over time

Use the past period data for given DMU and current period data for all others

DS	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
D1	0.891	1	0.966	0.943	0.975	0.837	1	1	1	1	1	1	0.947	1	1	1	1
D2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
D5	1	1	1	1	1	1	1	1	1	1	0.940	1	0.990	1	1	1	1
D6	0.910	0.889	0.955	0.809	0.940	0.805	0.768	0.819	0.853	0.803	0.926	0.948	0.905	0.901	0.920	0.865	0.859

2. Forecasting Based Approaches

2.1 Forecasting Future Performance

Use last period data for D1 and forecast the next period data for others

Measure the performance of D1 with respect to the future behavior of others

Drug Stores	Efficiency Score	Inputs						Outputs
		Energy	Rent	Admin	Material	Wages	Cost	Profit
D1	1	12743	123250	15658	1566000	97250	1814901	240750
		0	0.0039	0	0	0.0031	0.0001	0.0034
D2	1	6847	110000	11625	902750	83735	1114957	188517
		0	0	0	0	0	0.0009	0.0045
D3	1	8138	100375	12139	712150	54950	887751	143499
		0	0	0	0	0	0.0011	0

4 year Moving Average/VRS-I Model

2.2 Forecasting Future Targets

Use last period targets for D1 and forecast the next period targets for others

Setting targets according to the forecasted targets

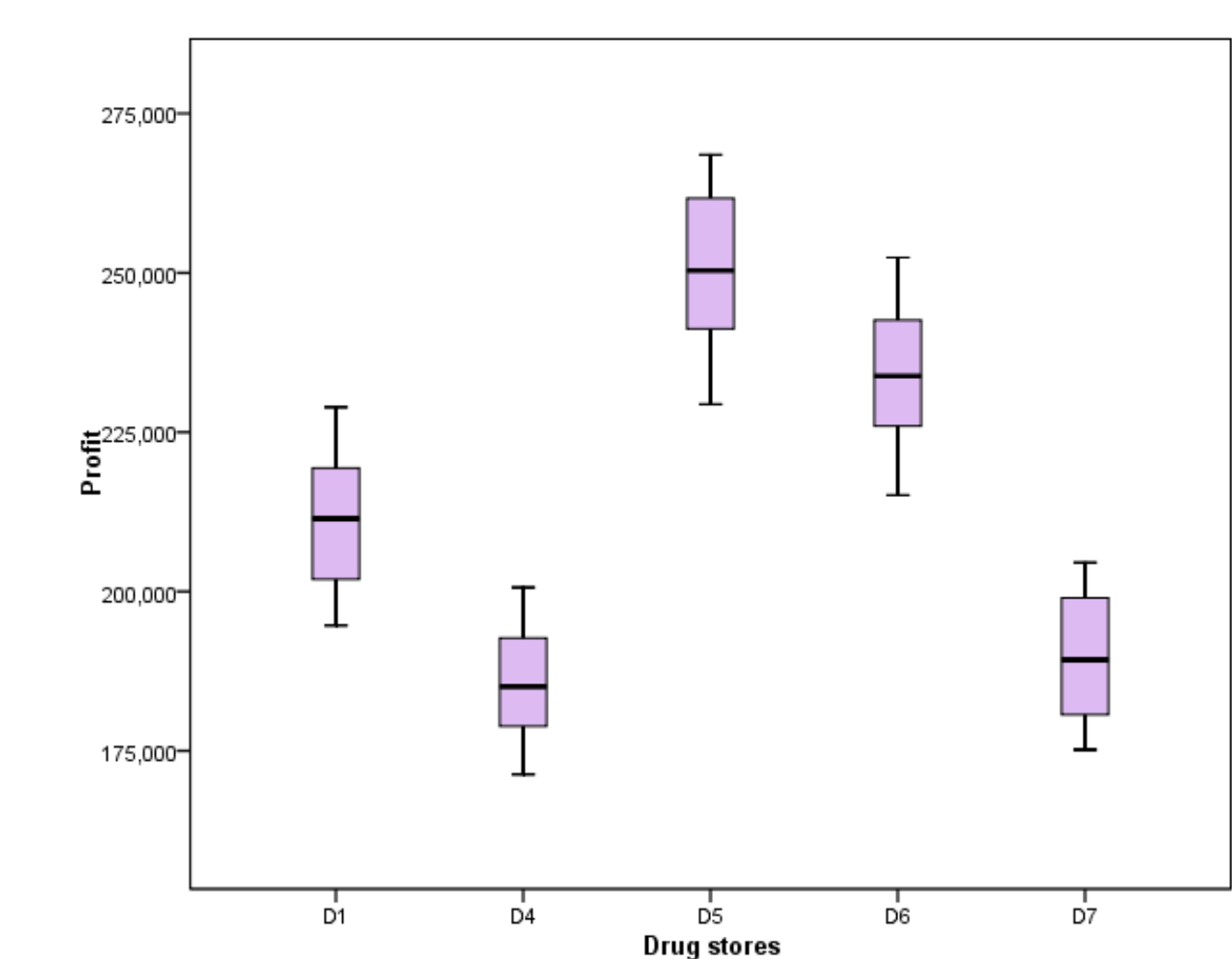
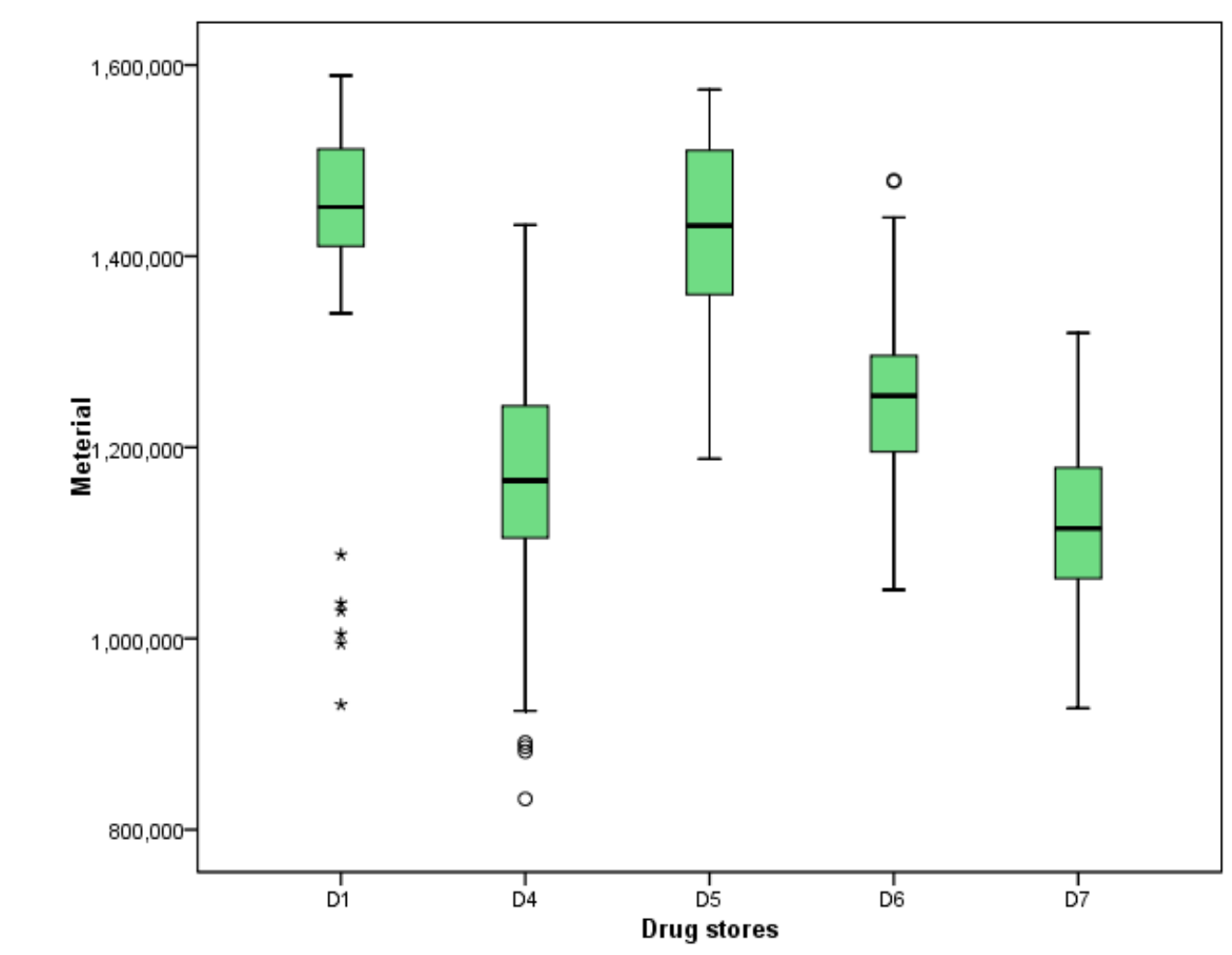
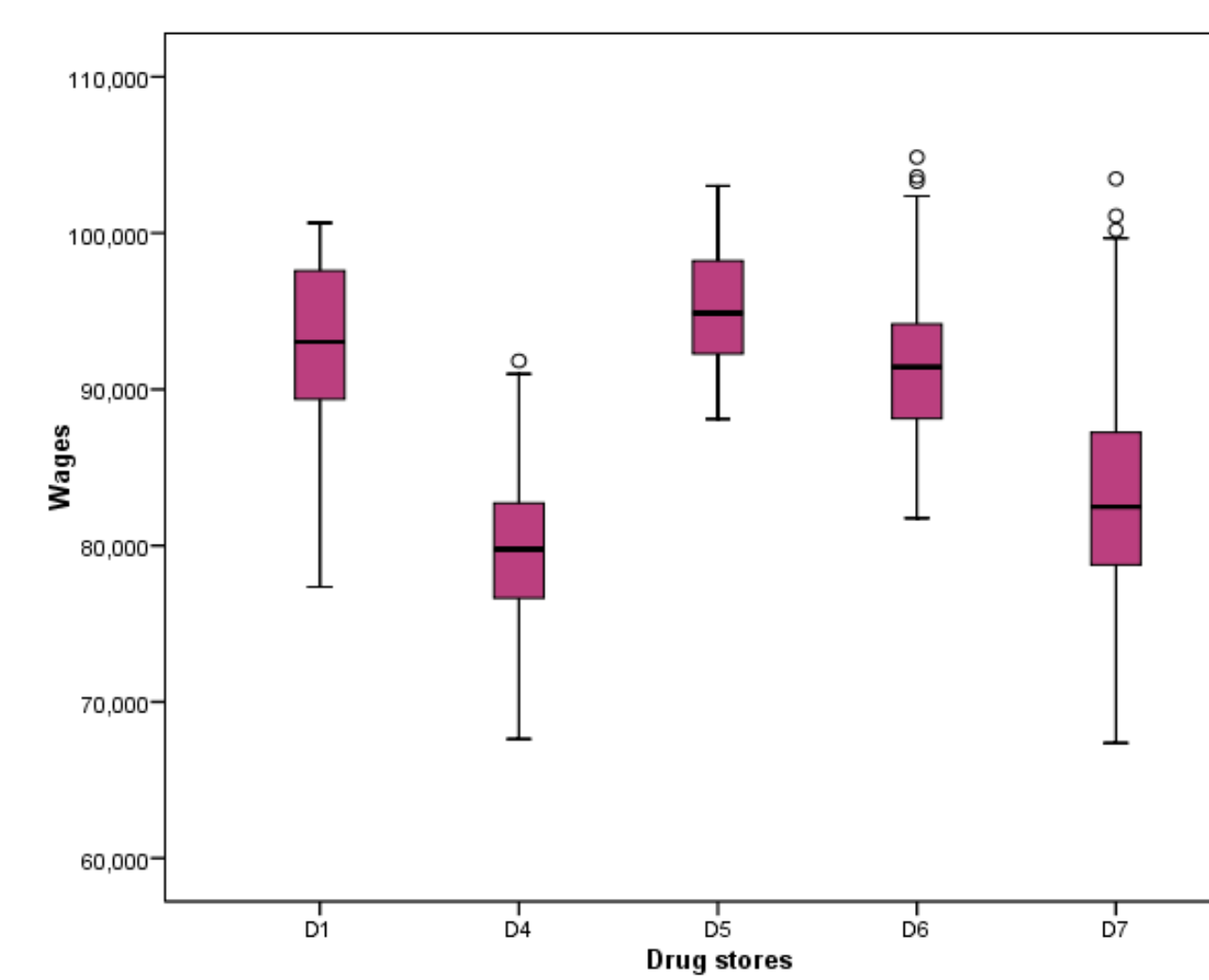
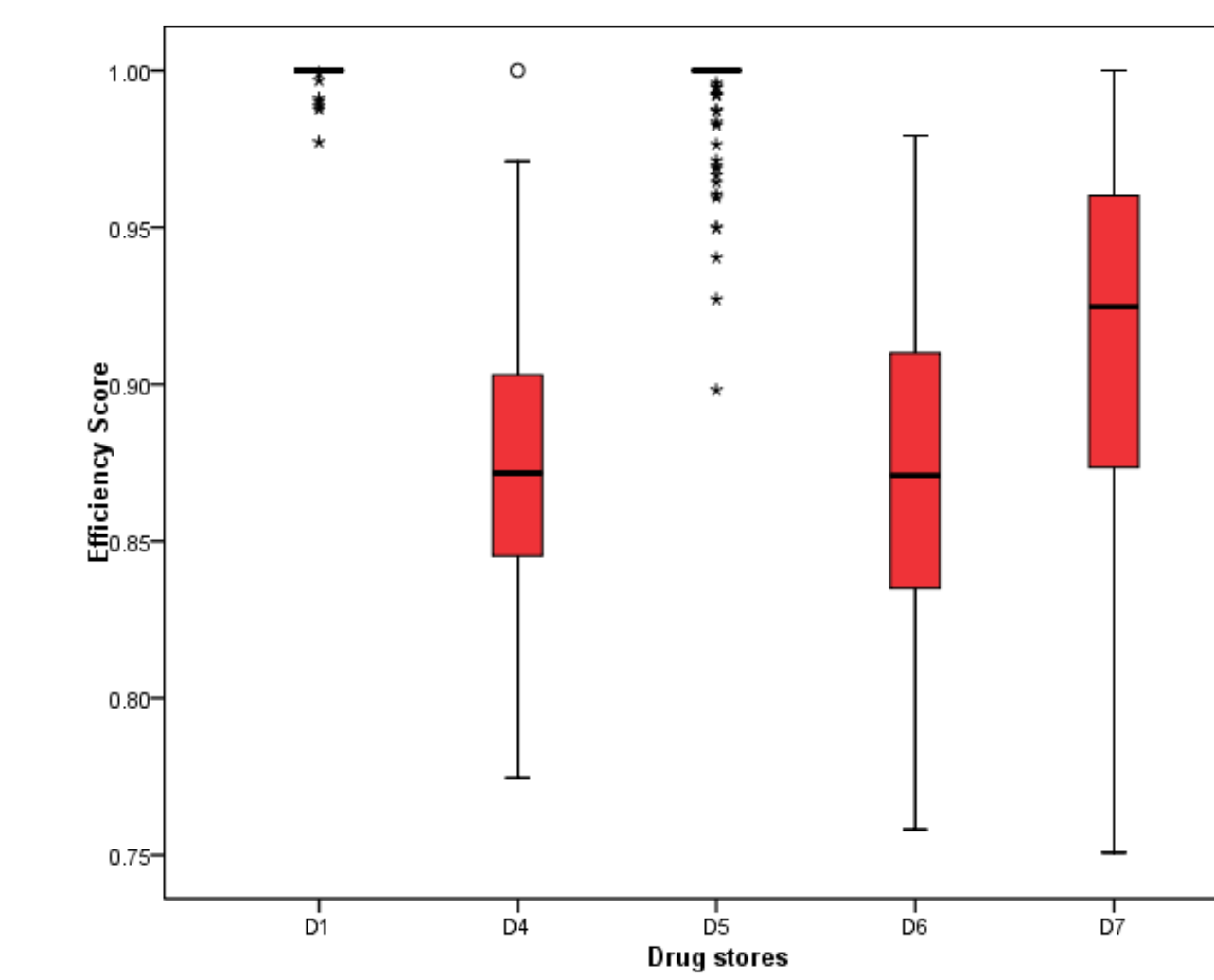
Drug Stores	Efficiency Score	Inputs						Outputs
		Energy	Rent	Admin	Material	Wages	Cost	Profit
D1	1	11356	130000	11560	1590000	101100	1844016	229000
		0	0.00436	0.01118	0	0.00300	0	0.00343
D2	1	6847	110000	11625	902750	83735	1114957	188517
		0	0	0	0	0	0.00090	0.00453
D3	1	8137	100375	12138	712150	54950	887751	143499
		0	0	0	0	0	0.00113	0

4 year Moving Average/VRS-I Model

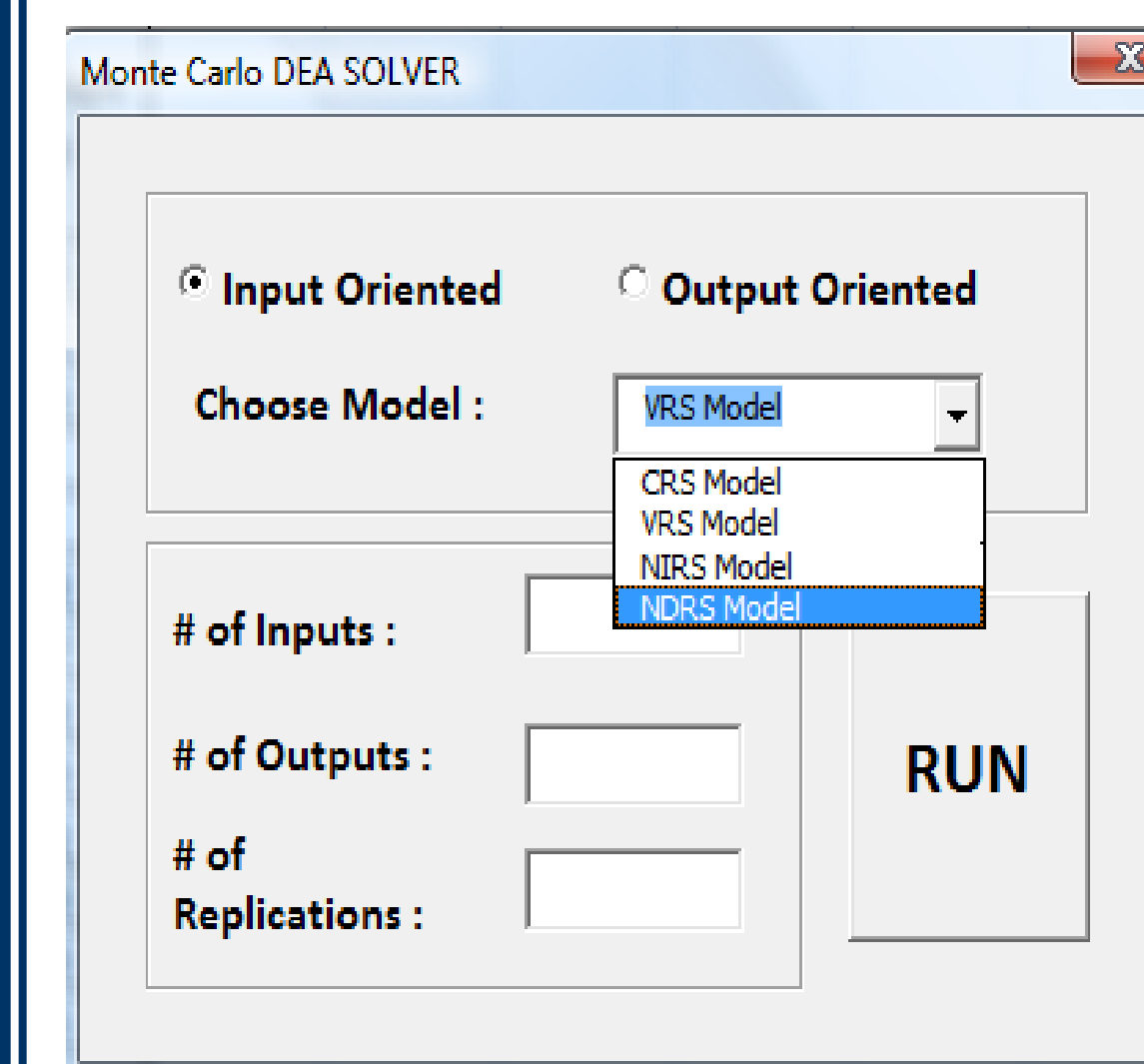
RESULTS (cont'd)

3. Monte Carlo Approach

Scenarios	Data	Data Generation
Scenario 1	Last period data	Inputs and Outputs improved randomly
Scenario 2	Last period data	Inputs and Outputs improved or worsened randomly
Scenario 3	Last period targets	Input and output targets of last period improved randomly
Scenario 4	Last period targets	Input and output targets of last period improved or worsened randomly
Scenario 5	Simple average of previous targets	Average of input and output targets improved randomly
Scenario 6	Simple average of previous targets	Average of input and output targets improved or worsened randomly



TOOL DEVELOPMENT



DMU	% Efficient	Statistics	Score	% of Prior Commitments	Inputs						Outputs								
					Cost per Case	Percent Felony	RacAdvis m Rate	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10		
Special Intensive Groups	100.00%	LB	1	16.47	1110	63.82	58.83684												
		UB	1	20.72	1110	68.99	64.16309												
		ave	1	18.61	1110	66.45	61.47416												
		std	0	1.221235	0	1.4985	1.563964												
		median	1	18.61	1110	66.46	61.48981												
TRY Centers	99.00%	LB	0.8884	15.33	1949	70.66	64.64932												
		UB	1	19.47	2243	82.75	69.76622												
		ave	0.999672	17.43	2242	80.49	67.17065												
		std	0.004361	1.199906	12.85	1.4581	1.487045												
		median	1	17.44	2243	80.47	67.20087												
Assoc. Marine Institutes	54.70%	LB	0.8175	15.42	1394	88.03	64.73237												
		UB	1	21.35	3735	85.49	69.86453												
		ave	0.962872	18.39	3048	80.32	67.2392												
		std	0.048506	1.388753	774.19	4.1540	1.501352												
		median	1	18.08	3735	81.90	67.22489												

Publications

- Ceyhan ME, Benneyan JC (2009). "Longitudinal Targets in Data Envelopment Analysis" *Industrial Engineering Research Conference*, accepted for publication.
- Benneyan, J.C, Sunnetci, A., Ceyhan, M.E. (2008). Data Envelopment Analysis Models for Identifying and Benchmarking the Best Healthcare Processes. *International Journal of Six Sigma*, 4(3), 305-331.