

DAMAGE LOCALIZATION IN OUTPUT-ONLY SYSTEMS: A FLEXIBILITY BASED APPROACH

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ABSTRACT

To assemble flexibility matrices from vibration signals the input must be measured and that there must be, at least, one co-located sensor actuator pair. Techniques to localize damage that use changes in flexibility have not, therefore, found application in the important case of ambient excitation. The central topic of this paper is the discussion of an approach that allows extension of flexibility based damage localization to cases where only output measurements are available. The fundamental idea is that lack of deterministic information on the input can be partially compensated by knowledge of the structure of the mass matrix. In particular, when the inverse of the mass matrix in the coordinates defined by the sensors can be assumed diagonal then matrices that differ from the flexibility by a scalar multiplier can be assembled and these matrices can replace the flexibility with no loss of useful information.

NOMENCLATURE

F_U ,	undamaged flexibility matrix
F_D ,	damaged flexibility matrix
DF	change in flexibility matrix
DLV	damage locating vector
nsi	normalized stress index
WSI	weighted stress index
M	mass matrix
K	stiffness matrix
C	damping matrix
Ψ	displacement partition of the complex modes
Ψ_m	complex modes at sensor locations
d_g^{-1}	normalization constants
m	number of output sensors
n	number of modes

INTRODUCTION

When experimental data is used to improve a mathematical model the parameters that are candidates for updating are typically selected with guidance from a priori knowledge on what aspects of the model are most uncertain. In the damage identification problem, however, uncertainty or sensitivity can not be used as criteria to arrive at a set of free parameters. Since the use of a large parameter space leads to ill-conditioning and non-uniqueness methods that can provide objective information about damage without the need to refer to a detailed model of the structure are of considerable practical importance. Among the techniques that attempt to localize damage without reference to a model, those that operate with changes in mode shapes have received significant attention [1,2,3]. A difficulty often faced when using mode shape changes to locate damage, however, is the fact that the appropriate pairing of modes from the reference to the potentially damaged-state is not always apparent. One way to get around this difficulty is to use all the available modes to assemble flexibility matrices and then focus on the change of these matrices [4,5]. The fact that the flexibility is dominated by the lower modes (which are typically the ones that can be identified experimentally) and that it can be assembled at whatever sensor coordinates are available are convenient features of the approach.

An important limitation on the use of flexibility is the fact that these matrices can only be assembled from the data when the input is measured [6,7]. Since in civil engineering structures a full characterization of the input is often impractical, the flexibility-based damage localization has not been considered as a viable option in buildings or bridges. This paper shows, however, that while it is true that flexibility matrices can

not be assembled from vibration data for output only systems, matrices that differ from the flexibility by a single scalar multiplier can be obtained when some conditions prevail. Specifically, the paper shows that matrices that are proportional to the flexibility can be computed exclusively from the measured data if the inverse of the mass matrix can be assumed diagonal over the coordinates defined by the available sensors. The objective of this paper is to discuss the issues associated with the computation of the flexibility proportional matrices and to illustrate how these matrices can be used in damage localization applications.

The paper is organized as follows. The first section presents a summary of a recently developed technique for interrogating changes in flexibility about damage localization. The technique, designated as the Damage Locating Vector Approach, (DLV) locates the damage by inspecting stress fields created by vectors that are contained in the null space of the change in flexibility [8]. The next section illustrates how modal orthogonality can be combined with knowledge of the structure of the inverse of the mass matrix to arrive at matrices that differ from the flexibility by a single scalar multiplier. This development is made for a viscously damped system but no limitation on the nature of the damping (classical or not classical) is introduced. The theoretical part of the paper concludes with a discussion on how to ensure that the missing scalar in the flexibility matrix, computed for the reference and the damaged states, are essentially the same. A numerical example on a 4 DOF system illustrates the techniques discussed.

THE DLV TECHNIQUE

The Damage Locating Vector (DLV) approach provides a systematic way for interrogating changes in flexibility matrices with respect to the localization of the damage. In this section the basic features of the technique are reviewed. A more detailed discussion of the theoretical background, as well as discussion on robustness and other issues may be found in [8]. As shall be evident from the results in this section, a missing scalar multiplier in the flexibility matrices is immaterial in the DLV localization, provided the scalar can be made the same in the reference and the damaged states.

The basic idea in the DLV approach is that the vectors that span the null-space of the change in flexibility (from the undamaged to the damaged states) when treated as static loads on the system, lead to stress fields that are zero over the damaged elements. Depending on the number and location of the sensors

the intersection of the null stress regions identified by the DLVs may or may not exclusively contain damaged elements. Elements that are undamaged but which cannot be theoretically discriminated from the damaged ones by changes in flexibility (for a given set of sensors) are designated as inseparable. The steps of the DLV localization can be summarized as follows:

1. Compute the change in flexibility as;

$$DF = F_U - F_D \quad (1)$$

2. Obtain a singular value decomposition of DF , namely;

$$DF = U \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix} V^T \quad (2)$$

where s_2 are 'small' singular values. For ideal conditions s_2 contains zeros and the DLV vectors are simply the columns of V associated with the null space. For the noisy conditions that prevail in practice, however, the values in s_2 are never equal to zero and a cutoff has to be established to select the dimension of the null space. The vectors in V that can be treated as DLVs for noisy conditions can be selected as follows (see ref. [8] for the mathematical support)

- a) Compute the stresses in an undamaged model of the structure using the columns in V as loads.

- b) Reduce the internal stresses in every element (for a given load vector) to a single characterizing stress, σ (strain energy per unit characterizing dimension should be proportional to σ^2).

- c) Designate the reciprocal of the maximum value of the characterizing stress as c_j . Compute the svn index for every vector in V as;

$$svn_j = \sqrt{\frac{s_j c_j^2}{s_q c_q^2}} \quad (3)$$

where;

$$s_q c_q^2 = \max (s_j c_j^2) \quad \text{for } j=1: m \quad (4)$$

The vectors for which $svn \leq 0.20$ can be treated as DLVs. Once the set of DLV vectors has been identified the localization proper is carried out as follows:

3. Compute, for each DLV vector, the normalized stress index vector as;

$$nsi_i = \frac{\sigma_i}{\sigma_i|_{max}} \quad (5)$$

4. Compute the vector of weighted stress indices, WSI , as;

$$WSI = \frac{\sum_{i=1}^{ndl} nsi_i}{ndl} \quad (6)$$

where $\overline{svn}_i = \max(svn_i, 0.015)$ and ndl is the number of DLV vectors. The potentially damaged elements are those having $WSI < 1$.

FLEXIBILITY MATRICES TO WITHIN A SCALAR MULTIPLIER

This section develops expressions for the inverse of the mass and the stiffness (the flexibility) in terms of the eigenvalues of the system and the corresponding eigenvectors at the sensor coordinates. It is assumed, for generality, that the damping is not classical so the modes are irreducibly complex. The expressions obtained for M^{-1} and K^{-1} are valid for a particular scaling of the modes which, as shall be shown, can only be enforced to within a missing scalar when the input is not measured.

We begin by considering the homogeneous equation for a time invariant finite-dimensional linear system with viscous dissipation, namely;

$$M\ddot{x} + C\dot{x} + Kx = 0 \quad (7)$$

where M , C and $K \in \mathfrak{R}^{n \times n}$ are the mass, damping and stiffness matrices. For a state vector defined using displacements and velocities the first order form of eq.7 that preserves symmetry is;

$$E \dot{y} = G y \quad (8)$$

where,

$$E = \begin{bmatrix} C & M \\ M & 0 \end{bmatrix}, \quad G = \begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix}, \quad y = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \quad (9a,b,c)$$

Assuming a solution $y = \phi e^{\lambda t}$ eq.8 leads to the eigenvalue problem;

$$E \phi \lambda = G \phi \quad (10)$$

which, since the matrices are real, yields real or complex conjugate eigenvalues λ . Assuming that the

system has a full set of eigenvectors (repeated eigenvalues are permitted) the solutions to eq.10 can be organized as;

$$\Phi_E = \begin{bmatrix} \Psi & \Psi^* \\ \Psi \Lambda & \Psi^* \Lambda^* \end{bmatrix}, \quad \Lambda_E = \begin{bmatrix} \Lambda & \\ & \Lambda^* \end{bmatrix} \quad (11a,b)$$

where

$$\begin{bmatrix} \Psi \\ \Psi \Lambda \end{bmatrix} = \Phi = [\phi_1 \quad \phi_2 \quad \dots], \quad \text{and } \Lambda = \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \end{bmatrix} \quad (12a,b)$$

and the superscript * stands for complex conjugate.

The symmetry of E and G can be exploited to show that;

$$\Phi_E^T E \Phi_E = \begin{bmatrix} d_e & \\ & d_e^* \end{bmatrix}^{-1} \quad (13)$$

$$\Phi_E^T G \Phi_E = \begin{bmatrix} d_g & \\ & d_g^* \end{bmatrix}^{-1} \quad (14)$$

From where it follows that;

$$E^{-1} = \Phi_E \begin{bmatrix} d_e & \\ & d_e^* \end{bmatrix} \Phi_E^T \quad (15)$$

$$G^{-1} = \Phi_E \begin{bmatrix} d_g & \\ & d_g^* \end{bmatrix} \Phi_E^T \quad (16)$$

Equating the inverse of the matrices E and G in eq.9a and b with the results in eqs.15 and 16 one gets a number of equalities of which we list the ones that are relevant for our purposes:

$$M^{-1} = 2 \Re (\Psi \Lambda d_g \Lambda^T \Psi^T) \quad (17)$$

$$M^{-1} = 2 \Re (\Psi d_e \Lambda^T \Psi^T) \quad (18)$$

$$K^{-1} = -2 \Re (\Psi d_g \Psi^T) \quad (19)$$

$$\Re (\Psi \Lambda d_g \Psi^T) = 0 \quad (20)$$

$$\Re (\Psi d_e \Psi^T) = 0 \quad (21)$$

From eqs. 17 and 18 one concludes that;

$$\Lambda d_g = d_e \quad (22)$$

and thus the previous expressions are reduce to;

$$M^{-1} = 2 \Re (\Psi \Lambda^2 d_g \Psi^T) \quad (24)$$

$$K^{-1} = -2\Re(\Psi d_g \Psi^T) \quad (25)$$

$$\Re(\Psi \Lambda d_g \Psi^T) = 0 \quad (26)$$

The diagonal matrix d_g can be computed from the data without knowledge of the system matrices in the deterministic input case. In the stochastic case, however, this is not possible and one must introduce some apriori knowledge to proceed. Consider as an introduction the case where there is enough information about the mass matrix to allow the computation of the partition of the inverse over the sensor coordinates. If there are m output sensors then one can use eqs.24 and 26 to set up $m(m+1)$ equations to solve for the complex constants d_g . Assuming all the modes are identified one can easily show that the number of sensors required to identify the normalizing constants has to satisfy $m \geq 0.5(\sqrt{8n+1} - 1)$ where n is the number of modes (order is $2n$). For example, if there are 10 modes in the system the normalizing constants can be obtained from knowledge of the inverse of the mass if $m \geq 4$. Note that if only a truncated modal space is available then the approach would lead to some approximation in the constants because one would be equating a 'converged physical quantity' (the left side of eq.24) with a modally truncated approximation.

In this paper we pursue a less restrictive assumption than that of presuming complete knowledge of M^{-1} . In particular, we proceed by assuming simply that M^{-1} is diagonal. It is evident, of course, that what we're doing is saying that 'we know' the off-diagonal part of M^{-1} to be zero. In this case we lose m equations and, because the equations are now all equal to zero we can compute the normalizing factors only up to r undetermined constants, where r is the nullity of the resulting coefficient matrix. It is possible to show that a necessary condition for the nullity to be one (the smallest it can be) is that the number of sensors satisfy $m \geq \sqrt{2n}$. So, for $n = 10$, for example, one needs at least 5 sensors. The comment made previously with respect to equating a physical quantity (in this case the zeros in the off diagonals of M^{-1}) to a truncated approximation holds without modification in this case also. The specifics used to arrive at the normalization constants in the case where M^{-1} is assumed diagonal are described next.

Define;

$$H_R + H_I i = \Psi_{m,j} \Psi_{m,j}^T \Lambda_j^2 \quad (27)$$

Eq.24, therefore, can be written in the domain of real numbers as

$$M^{-1} = \sum_{j=1}^n (H_{R,j} d_{gR,j} - H_{I,j} d_{gI,j}) \quad (28)$$

where d_{gR} and d_{gI} are the real and the imaginary components of d_g . Taking the upper (lower) triangular portion of M^{-1} (without the main diagonal) and placing it in vector form one can write

$$0 = \sum_{j=1}^n (\hat{H}_{R,j} d_{gR,j} - \hat{H}_{I,j} d_{gI,j}) \quad (29)$$

where the order of the entries in the vectors \hat{H}_j is arbitrary as long as one is consistent in defining $\hat{H}_{R,j}$ and $\hat{H}_{I,j}$. We now define the vector β as all the real components of d_g followed by all the imaginary components, namely

$$\beta = [d_{gR1} \ d_{gR2} \ \dots \ d_{gI1} \ d_{gI2} \ \dots]^T \quad (30)$$

With the preceding notation eq.29 can be written as;

$$\bar{H} \beta = 0 \quad (31)$$

where;

$$\bar{H} = [\hat{H}_{R,1}, \hat{H}_{R,2}, \dots - \hat{H}_{I,1}, -\hat{H}_{I,2}, \dots] \quad (32)$$

It follows then that β (which contains the required normalizing constants ordered as per eq.30) is in the null space of the matrix \bar{H} . One can further restrict the subspace that contains β by taking advantage of the relationship in eq.26. Indeed, following the same approach used to pass from eq.29 to eq.31 one finds that eq.26 gives;

$$\bar{S} \beta = 0 \quad (33)$$

where the only difference is that the assembly of \bar{S} includes the diagonal of the matrix in eq.26. Combining eqs.31 and 33 one gets;

$$\begin{bmatrix} \bar{H} \\ \bar{S} \end{bmatrix} \beta = Y \beta = 0 \quad (34)$$

So β is in the null space of Y . If all the modes are available, the nullity of Y is one and β can be computed to within a single scalar. The complex constants d_g (to within a scalar) are then given by eq.30. and the flexibility proportional matrix is evaluated with eq.25. In practice, of course, one

seldom obtains 'all the modes' and, as a result, the matrix Y in eq.34 proves to be full rank. An approximate solution can be obtained by taking β as the singular vector associated with the smallest singular value of Y .

COMPATIBILITY OF THE SCALAR MULTIPLIER

The scalar that is missing in the flexibility matrices of the previous section is arbitrary and is not necessarily the same for the undamaged and the damaged states. In order to preserve the null space when taking the difference of the flexibility proportional matrices it is necessary to ensure that the missing constant is consistent. Two procedures have been examined thus far for ensuring compatibility. The first one is based on the idea that the mass matrix has not changed as a result of the damage. If this is true and the modal space is 'complete' then the scaling factor can be adjusted so that the inverse of the mass in the two states (as given by eq.28) are the same. Of course, in practice approximations are inevitable and one can not pretend that it will be possible to make the two M^{-1} expressions identical. One can, however, define a norm and adjust the scaling to make this norm the same in the two states.

When all the modes are not available the contribution of the available modes to M^{-1} may differ in the two states and one can not argue that the error in making the missing scalar compatible derives exclusively from round off and imprecision. Thus far we have looked at two procedures for attaining approximate compatibility between the flexibility proportional matrices of the reference and the damaged states. The first approach uses the trace of the M^{-1} as the metric that should be equal in the two states and the second, which does not use information in M^{-1} , is outlined next.

Assume that the undamaged and damaged flexibility proportional matrices are \bar{F}_U and \bar{F}_D while the true modally truncated but, undetermined matrices, are F_U and F_D . One can then write;

$$\bar{F}_U = \alpha F_U \quad (35)$$

and

$$\bar{F}_D = \eta F_D \quad (36)$$

where α and η are undetermined constants. Assume L is a vector in the 'effective null space' of the modally truncated true change in flexibility. Recognizing that $F_U L \cong F_D L \cong d$ one can write (replacing the \cong with = for simplicity);

$$\bar{F}_U L = \alpha d \quad (37)$$

and

$$\bar{F}_D L = \eta d \quad (38)$$

Solving for L in eq.37 and substituting the result in eq.38 one gets;

$$\bar{F}_U^{-1} \bar{F}_D d = \frac{\eta}{\alpha} d \quad (39)$$

which shows that the desired η/α ratio is a real eigenvalue of the matrix on the left side.

Since there are several real eigenvalues, however, the solution does not immediately point out which is the correct result. An approach that helps in reducing ambiguity is to first normalize the matrices \bar{F}_U and \bar{F}_D in such a way that the η/α ratio is necessarily less than 1. This is the case, for example, if the flexibility proportional matrices are normalized to equal trace because the true value of the trace in the damaged flexibility is larger (assuming that the effect of damage is more than the inevitable error in the computations). If this scaling is first introduced then one can discard not only the η/α ratios that are complex but also those that are greater than 1. Unfortunately, there appear to be cases where the solution is still not unique after this is done.

While research on the compatibility of the scalar continues, the limited numerical experience gained thus far suggests that the simple approach based on the trace of M^{-1} may be sufficiently accurate.

NUMERICAL EXAMPLE

The basic steps of damage localization using the DLV approach for unmeasured input are illustrated using the simple system shown in fig.1. Modal truncation and error in the identification are not contemplated in this example.

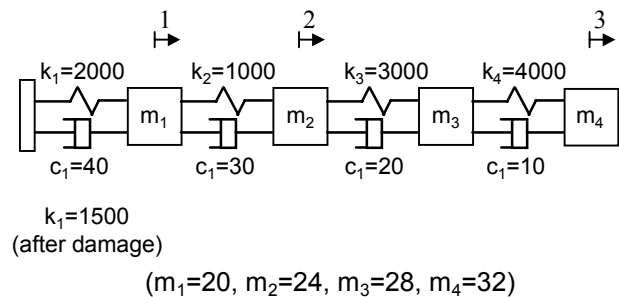


Figure 1 System considered

Output sensors exist at coordinates 1, 2 and 3 and damage is simulated as 25% reduction in the stiffness of spring k_1 .

The first step is to extract arbitrarily scaled complex mode shapes Ψ_m at the sensor coordinates. This can be done using any suitable stochastic identification algorithm [9]. For the system in fig.1 the noise free results in the undamaged state are;

$$\begin{bmatrix} -0.024+0.058i & -0.059-0.030i & -0.047+0.033i & -0.001-0.006i \\ -0.067+0.167i & -0.035-0.036i & 0.020-0.033i & 0.018+0.016i \\ -0.084+0.204i & 0.029+0.027i & -0.012+0.019i & 0.018+0.009i \end{bmatrix}$$

The second step is to obtain the normalization constants from eq.34. This requires that one assemble \bar{H} and \bar{S} . The computations for the first mode are illustrated in the following. Assembling H_1 from eq.27 one gets;

$$H_1 = \begin{bmatrix} 0.017+0.020i & 0.051+0.057i & 0.062+0.071i \\ 0.051+0.057i & 0.149+0.161i & 0.180+0.199i \\ 0.062+0.071i & 0.180+0.199i & 0.219+0.247i \end{bmatrix}$$

The upper triangular part of the matrix (without the main diagonal) contains, in this case, 3 numbers. The real and the imaginary components of these numbers are used to form the vectors $\hat{H}_{R,1}$ and $\hat{H}_{I,1}$, namely;

$$\hat{H}_{R,1} = \begin{bmatrix} 0.051 \\ 0.062 \\ 0.180 \end{bmatrix} \quad \hat{H}_{I,1} = \begin{bmatrix} 0.057 \\ 0.071 \\ 0.199 \end{bmatrix}$$

where the order selected is arbitrary. Repeating these steps for all the modes one obtains the vectors needed to form \bar{H} (see eq.32). The result is;

$$\begin{bmatrix} 0.051 & -0.072 & 0.073 & -0.034 & -0.057 & 0.381 & 0.399 & -0.041 \\ 0.062 & 0.071 & -0.047 & -0.018 & -0.071 & -0.299 & -0.229 & -0.040 \\ 0.180 & -0.021 & -0.031 & -0.044 & -0.199 & -0.234 & 0.154 & 0.174 \end{bmatrix}$$

where the columns of \bar{H} corresponding to the first mode are highlighted. The same approach is repeated to form \bar{S} . One gets;

$$S_1 = \begin{bmatrix} 0.008-0.007i & 0.021-0.020i & 0.026-0.025i \\ 0.021-0.020i & 0.060-0.059i & 0.074-0.072i \\ 0.026-0.025i & 0.074-0.072i & 0.092-0.087i \end{bmatrix}$$

Placing the results in vector form gives;

$$\hat{S}_{R,1} = \begin{bmatrix} 0.008 \\ 0.021 \\ 0.060 \\ 0.026 \\ 0.074 \\ 0.092 \end{bmatrix} \quad \hat{S}_{I,1} = \begin{bmatrix} -0.007 \\ -0.020 \\ -0.059 \\ -0.025 \\ -0.072 \\ -0.087 \end{bmatrix}$$

where we note that the main diagonal is also included since it is also zero according to eq.26. The matrix \bar{S} is assembled by repeating the process for all the modes, the results is;

$$\begin{bmatrix} 0.008 & -0.039 & 0.040 & -0.000 & 0.007 & -0.025 & -0.020 & 0.001 \\ 0.021 & -0.035 & -0.030 & 0.002 & 0.020 & -0.009 & 0.002 & -0.002 \\ 0.060 & -0.027 & 0.020 & -0.011 & 0.059 & 0.002 & 0.007 & -0.000 \\ 0.026 & 0.027 & 0.017 & 0.002 & 0.025 & 0.008 & -0.001 & -0.001 \\ 0.074 & 0.022 & -0.011 & -0.009 & 0.072 & -0.001 & -0.004 & -0.003 \\ 0.092 & -0.017 & 0.006 & -0.006 & 0.087 & -0.000 & 0.002 & -0.004 \end{bmatrix}$$

Combining \bar{H} and \bar{S} one obtains the matrix Y . From inspection of the singular values, s , it is evident that the nullity is one. The singular values s and the nullspace β are shown below.

$$s = \begin{bmatrix} 0.684 \\ 0.432 \\ 0.191 \\ 0.117 \\ 0.073 \\ 0.053 \\ 0.009 \\ 7.18E-17 \end{bmatrix} \quad \beta = \begin{bmatrix} -0.3232 \\ 0.1191 \\ 0.0743 \\ 0.1491 \\ 0.3386 \\ -0.5149 \\ 0.5603 \\ -0.3999 \end{bmatrix}$$

From β one readily gets the complex constants, d_g from eq.30. The inverse of the mass and stiffness matrices can then be calculated from eqs.24 and 25. One gets;

$$M_U^{-1} = \alpha \begin{bmatrix} 0.41 & 0.00 & 0.00 \\ 0.00 & 0.34 & 0.00 \\ 0.00 & 0.00 & 0.25 \end{bmatrix} \quad K_U^{-1} = \alpha \begin{bmatrix} 0.004 & 0.004 & 0.004 \\ 0.004 & 0.012 & 0.012 \\ 0.004 & 0.012 & 0.017 \end{bmatrix}$$

The exact values for these matrices are;

$$M_e^{-1} = \begin{bmatrix} 0.050 & 0.000 & 0.000 \\ 0.000 & 0.042 & 0.000 \\ 0.000 & 0.000 & 0.031 \end{bmatrix} \quad K_e^{-1} = \begin{bmatrix} 0.050 & 0.050 & 0.050 \\ 0.050 & 0.150 & 0.150 \\ 0.050 & 0.150 & 0.208 \end{bmatrix} \times 10^{-2}$$

from where it can be seen that the difference is a simple scalar multiplier.

Following the same steps outlined previously the system matrices extracted for the structure in the damaged state are;

$$M_D^{-1} = \alpha \begin{bmatrix} 0.41 & 0.00 & 0.00 \\ 0.00 & 0.34 & 0.00 \\ 0.00 & 0.00 & 0.25 \end{bmatrix} \quad K_D^{-1} = \alpha \begin{bmatrix} 0.005 & 0.005 & 0.005 \\ 0.005 & 0.014 & 0.014 \\ 0.005 & 0.014 & 0.018 \end{bmatrix}$$

where the trace of M^{-1} has been normalized to unity in both cases to ensure compatibility of the missing scalar. Once the flexibility proportional matrices are calculated within a common multiplier, the SVD of the change in these matrices yields the singular values and right side singular vectors shown.

$$s = \begin{bmatrix} 0.0041 \\ 5.41 \times 10^{-17} \\ 3.44 \times 10^{-18} \end{bmatrix} \quad V = \begin{bmatrix} 0.58 & -0.81 & 0.12 \\ 0.58 & 0.31 & -0.76 \\ 0.58 & 0.50 & 0.64 \end{bmatrix}$$

Inspection of the singular values shows that there are 2 DLVs. Applying the two vectors identified as DLVs as loads at the sensor coordinates one gets the axial force distribution N shown in fig2.

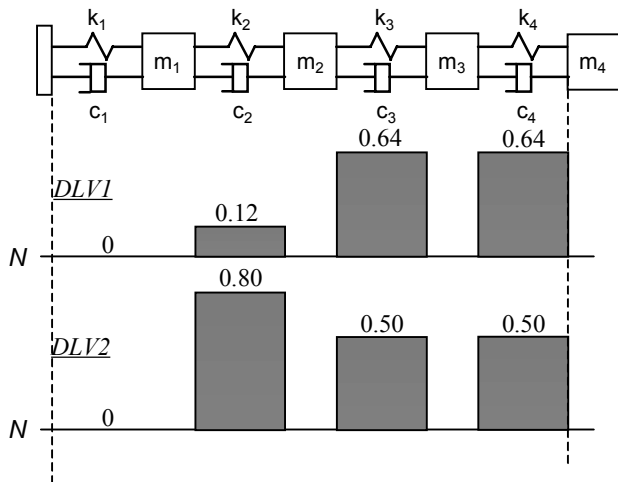


Figure 2 Axial force distribution

Both vectors, as expected, locate the damage correctly at the first spring.

CONCLUSIONS

This paper extends the DLV damage localization technique to the important case where the input can not be deterministically characterized. This is done by showing that flexibility proportional matrices can be extracted from the data without knowledge of the input and that the undetermined missing scalar can be made consistent for the reference and the damaged states.

The paper shows that computation of the flexibility proportional matrix is theoretically exact only when the modal basis is complete. Examination of the relationships involved suggests, however, that good approximations of the truncated flexibility should result when the truncated space provides a reasonable approximation for M^{-1} . The experience derived from numerical experiments has thus far supported this expectation.

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