OBSERVER/KALMAN AND SUBSPACE IDENTIFICATION OF THE UBC BENCHMARK STRUCTURAL MODEL

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Introduction

A two-bay by two-bay four story steel frame owned by the University of British Columbia has been selected as a benchmark structural model to test damage identification techniques. The structure is approximately in a 1:4 scale and has member sizes and details that may be found in Ventura et al. (1997). Testing to generate data for damage identification is scheduled to begin in the summer of 2000. Before addressing the full complexity associated with real experimental data, however, it was considered appropriate to examine a number of cases using simulated data. For this purpose, mathematical models of the test structure with various degrees of refinement have been formulated and used to generate data for various damage scenarios. This paper documents the damage identification approach used by the authors and presents the results obtained for the cases considered thus far.

Cases Considered

In all the cases examined the structure in both the undamaged and damage states is symmetric and loaded in such a way that the floor slabs undergo pure translation. Boundary accelerations are not imposed and since the weight of the floors is large in comparison to the weights of the columns and bracing elements, the dynamics are dominated by the four horizontal translations of the floors in each direction. Two damaged scenarios are considered: 1) removal of bracing elements on the first story and, 2) removal of bracing elements from the first and the third stories. Sensor noise is simulated by contaminating the analytically computed response with white noise having a RMS equal to 10% of that of the response. Two scenarios regarding available information are considered, namely: a) the input is measured and b) the input is not measured. Finally, two levels of refinement on the mathematical model used to generate the data are examined: 1) the data is generated using a shear building model (12-DOF) and, 2) the data is generated using a 120 DOF model (includes rotations and axial extensions). Complete details of the cases considered can be found in the paper by Johnson et. al. that appears in these proceedings.

Damage Identification Framework

The methodology presented in this section is formulated on the assumption that linearity holds in the pre and post-damage states. For the simulated data used here this condition is exactly realized.

Module 1 – Identification of Modal Characteristics

a) When the input is available the Eigensystem Realization Algorithm with a Kalman Observer is used (Juang 1994).
b) When the input is not measured Sub-ID, one of several subspace identification algorithms currently available (Van Overschee and Moor 1996) is applied.

Module 2 – Location of Damage Regions

This module contains two parts: a) computation of the flexibility matrix at the sensor locations and b) identification a subset of elements that contain the damaged elements.

a) Flexibility Matrix at Sensor Locations

Three techniques that apply when full sensor data is available are described in Bernal and Gunes (2000a). A brief description of one of them is presented next.

We begin by assuming that a realization has been obtained and we designate the eigenvalues and eigenvectors of the transition matrix A (in continuous time) as Λ and ψ, respectively. Given that the basis of the realization is unspecified, the entries in the eigenvectors reflect an undetermined combination of displacements and velocities and its necessary to perform a transformation to obtain the eigenvectors in the standard displacement-velocity form. The details of the transformation are well known and may be found in Bernal and Gunes (2000a), final result is;

$$\Phi_d = C\psi\ \Lambda^{-p}, \qquad \text{or} \qquad \Phi = \begin{bmatrix} C\psi\,[\Lambda^{-p}] \\ C\psi\,[\Lambda^{-p+1}] \end{bmatrix} \tag{1,2}$$

where p = 0, 1 or 2 for displacement, velocity or acceleration sensing respectively, Φ_d stands for the displacement partition of the eigenvector matrix and the matrix C is the state-to-output influence matrix obtained in the realization. The contribution of the identified modes to the flexibility matrix is;

$$F = \phi\ \left(\frac{d}{\omega}\right)^2 \phi^T \tag{3}$$

where ϕ is the matrix of undamped mode shapes, the term in parenthesis is a diagonal matrix where ω is the undamped natural frequency and d is a factor such that $\phi_i\ d_i$ is a mass normalized mode. If we assume that modal complexity is small then arbitrarily normalized undamped modes are obtained simply by rotating the identified complex modes of the realization. The mass normalization factor can be computed as follows (Bernal and Gunes 2000a):

$$d = (\tilde{Y}^T\tilde{Y})^{-1}\tilde{Y}^T\ \tilde{F} \tag{4}$$

where

$$Y = [I \quad I]\,S\,\Lambda\psi^{-1}B_z \qquad \Phi_d\ S^{-1} \approx \phi\,[I \quad I] \qquad F = \phi^T b_2 \tag{5,6,7}$$

and

$$\tilde{Y} = [diag(Y_1)\,diag(Y_2)...diag(Y_r)]^T \quad \tilde{F} = [F_1\ \ F_2\ \ ...\ F_r]^T \tag{8,9}$$

The previous mass normalization is restricted to cases where the input is measured. For stochastic input the normalization is carried out by assuming that the mass matrix is known.

b) *Localization of Damaged Regions*
As one anticipates, examination of the truncated flexibility can provide information on the spatial distribution of damage. In the cases considered here the model of the structure and the conditions considered are sufficiently simple that the region of damage can be identified by inspection. In general, however, this is not possible a systematic approach is necessary. A general approach to extract spatial information on the distribution of damage from changes in flexibility has been recently developed by the first author and is outlined next.

Consider a system for which the identified flexibility matrices in the pre and post damage states are *Fu* and *Fd*. Assume that there are a number of load distributions that produce identical deformations when applied to the undamaged and damaged systems. If we collect all the distributions that satisfy this requirement in the matrix L it is evident that one can write;

$$(Fu - Fd)L = 0 \tag{10}$$

which makes it apparent that L is the null left space of the change in flexibility resulting from damage (assuming $F_d - F_u \neq 0$). Performing a singular value decomposition of the incremental flexibility one can write;

$$(F_d - F_u) = [\tilde{q}_1 \quad L]\begin{bmatrix} \tilde{s}_1 & 0 \\ 0 & \approx 0 \end{bmatrix} q_2^T \tag{11}$$

where we have accounted for the fact that in practical applications the singular values associated with the left null space will not be exactly zero due to modal truncation or approximations in the identified eigenproperties. From a physical perspective one appreciates that the load distributions that induce no stress in the damaged elements belong to *L*. These type of vectors are here designated as Damage Locating Vectors (DLV) since they identify regions that contain damaged elements as regions of zero stress. It is important to emphasize that while elements with stresses from a DLV can be exclude from the 'free parameter space' the elements that show zero stress may or may not be damaged. The number of elements in the subset of potentially damaged elements identified by the DLV's decreases as the number of sensors increases.

It is possible to show that the number of vectors in *L* equals the number of sensors minus the rank of a stress resultant influence matrix *Q* where $Q_{i,j}$ is the stress resultant at damaged location j due to a unit value of a load at the ith sensor. Needless to say, the matrix *Q* is unknown when the identification is carried out since the damaged locations are undetermined. Nevertheless, the information is useful since it points out that there is a limit to the number of damaged elements that can be spatially identified with a given number of sensors. For example, in a truss where the damaged elements lead to a *Q* matrix of full rank there will be no DLV's when the number of damaged elements equals or exceeds the number of sensors. An examination of the effect of the number of identified modes and a complete documentation of other theoretical issues associated with the DLVs, as well as an illustration of their performance in a number of structural configurations will appear in a forthcoming paper. We note in closing this brief introduction that in the case of the shear buildings considered in this portion of the benchmark study the number of DLVs equals the number of sensors minus the number of damaged floors.

<u>Module 3 – Quantification of Damage (Model Updating)</u>
The results from modules 1 and 2 provide a subset of elements that contain the damaged ones. At this stage one may need to further discriminate between damaged and undamaged elements and to quantify the damage in the damaged components using a model update strategy. In the particular case of the benchmark structure, at the level of complexity considered thus far, one is attempting to identify damage in terms of story stiffness values and not at the level of elements. It is possible, therefore, to perform the model update by simply fitting a shear building model to the identified flexibility (using a least square criterion). In the damaged cases only the levels where the damage was identified are treated as free parameters.

Results

a) Table 1 lists the identified frequencies and damping ratios for all the cases considered. A comparison with the exact results (Johnson et. al.) shows excellent accuracy in all cases.
b) The absolute value of the story shears induced by the DLVs are added and plotted in Figs.1 and 2 for the cases considered. As can be seen from an inspection of this figure, the damaged levels are clearly identified.
c) The inter-story stiffness values obtained from the flexibility fit are presented in Table 2 together with the percent reduction.

Final Remarks

A multi-stage approach for damage identification, location and quantification has been presented and applied to a benchmark structure. The results obtained showed that the ERA-OKID technique and the Sub-Id approach were able to identify the modal characteristics accurately in all cases. A new technique based on a singular value decomposition of the change in flexibility has been described and utilized to identify the levels of the structure where the damage occurred. In the particular application of the decomposition of the change in flexibility has been described and utilized to identify the levels of the structure where the damage occurred. In the particular application of the benchmark structure the damage identification was restricted to the specification of the levels where damage was detected and to a quantification of the percent reduction in the inter-story stiffness resulting from the damage. In case 1 there is no modeling error because the data used in the identification was generated using a shear building. The results of the ID are in this case virtually exact. In case 2 there is modeling error in the sense that the data was generated using a model that has 120 DOF while the model update of the last stage is carried out assuming a shear building model. Accuracy in the computed

Table 1 Natural frequencies and damping ratios

CASE 1: Known Input							CASE 2: Known Input					
	No Damage		***Damage 1***		***Damage 2***		***No Damage***		***Damage 1***		***Damage 2***	
Mode	ξ(%)	f(Hz)	ξ (%)	f(Hz)	ξ (%)	f(Hz)	ξ(%)	f(Hz)	ξ(%)	f(Hz)	ξ(%)	f(Hz)
1	1.03	9.41	1.11	6.24	1.12	5.82	1.04	8.21	1.13	4.91	1.15	4.36
2	1.01	25.54	1.01	21.53	1.01	14.89	1.00	22.54	1.01	18.38	1.04	10.26
3	1.00	38.67	1.00	37.38	1.01	36.06	1.01	35.58	1.00	33.99	1.00	33.81
4	1.00	48.01	1.00	47.83	1.00	41.35	1.00	46.12	1.00	45.80	1.00	37.47
CASE 1: Unknown Input							**CASE 2: Unknown Input**					
	No Damage		***Damage 1***		***Damage 2***		***No Damage***		***Damage 1***		***Damage 2***	
Mode	ξ(%)	f(Hz)	ξ (%)	f(Hz)	ξ (%)	f(Hz)	ξ(%)	f(Hz)	ξ(%)	f(Hz)	ξ(%)	f(Hz)
1	1.05	9.42	0.99	6.24	1.13	5.80	1.25	8.20	1.15	4.91	1.02	4.31
2	1.15	25.54	0.81	21.55	0.76	14.89	0.92	22.55	0.87	18.39	1.38	10.27
3	1.00	38.67	0.96	37.38	1.13	36.08	1.22	35.55	0.96	33.95	0.94	33.78
4	1.00	47.89	0.94	47.71	1.00	41.41	0.90	46.10	0.85	45.79	1.21	37.51

CASE 3: Unknown Input													
	No Damage		***Damage 1***		***Damage 2***			***No Damage***		***Damage 1***		***Damage 2***	
Mode	ξ (%)	f (Hz)	ξ (%)	f (Hz)	ξ (%)	f (Hz)	**Mode**	ξ (%)	f (Hz)	ξ (%)	f (Hz)	ξ (%)	f (Hz)
1	1.23	9.41	0.97	6.23	1.08	5.79	**5**	0.94	38.66	0.88	37.44	1.19	36.14
2	1.10	11.83	1.21	9.91	1.28	9.51	**6**	0.97	48.09	0.86	47.34	1.02	41.33
3	1.02	25.53	0.85	21.52	0.94	14.91	**7**	1.11	48.48	1.16	47.94	1.14	46.80
4	1.35	31.97	0.95	28.90	0.84	24.90	**8**	0.97	60.19	0.98	60.03	0.91	54.25

Table 2 Interstory stiffnesses and % reductions

CASE 1 (known input)						CASE 2 (known input)				
	No Damage	***Damage 1***		***Damage 2***		***No Damage***	***Damage 1***		***Damage 2***	
Floor	K (×e7)	K (×e7)	% Δ	K (×e7)	% Δ	K	K (×e7)	% Δ	K (×e7)	% Δ
1	6.79	1.94	71.3	1.96	71.1	5.63e7	1.17	79.3	1.20	78.6
2	6.93	-		-	-	4.95e7	-	-	-	-
3	7.02	-		1.95	72.2	4.88e7	-	-	0.78	84.1
4	7.40	-		-	-	4.73e7	-	-	-	-
CASE 1 (unknown input)						**CASE 2 (unknown input)**				
	No Damage	***Damage 1***		***Damage 2***		***No Damage***	***Damage 1***		***Damage 2***	
Floor	K (×e7)	K (×e7)	% Δ	K (×e7)	% Δ	K(×e7)	K (×e7)	% Δ	K (×e7)	% Δ
1	6.79	1.96	71.1	1.96	71.2	5.76	1.18	79.8	1.18	79.5
2	6.76	-		-	-	4.75	-	-	-	-
3	6.93	-		1.94	72.0	4.77	-	-	0.77	83.9
4	6.84	-		-	-	4.51	-	-	-	-
CASE 3 x-direction						**CASE 3 y-direction**				
	No Damage	***Damage 1***		***Damage 2***		***No Damage***	***Damage 1***		***Damage 2***	
Floor	K (×e8)	K (×e7)	% Δ	K (×e7)	% Δ	K(×e8)	K (×e7)	% Δ	K (×e8)	% Δ
1	1.32	6.90	47.8	7.06	46.6	5.87	1.65	71.9	1.64	72.1
2	1.27	-		-	-	5.83	-	-	-	-
3	1.33	-		6.86	48.5	5.80	-	-	1.61	72.3
4	1.21	-		-	-	5.80	-	-	-	-

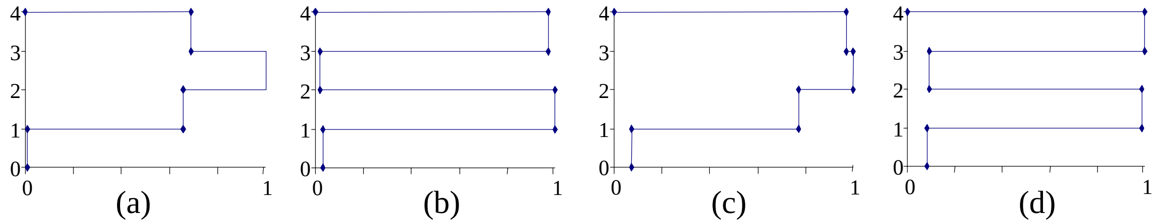

Figure 1 Story shears induced by DLVs for known input cases:
(a) Case 1-Damage 1 (b) Case 2- Damage 2 (c) Case 2- Damage 1 (d) Case 2- Damage 2

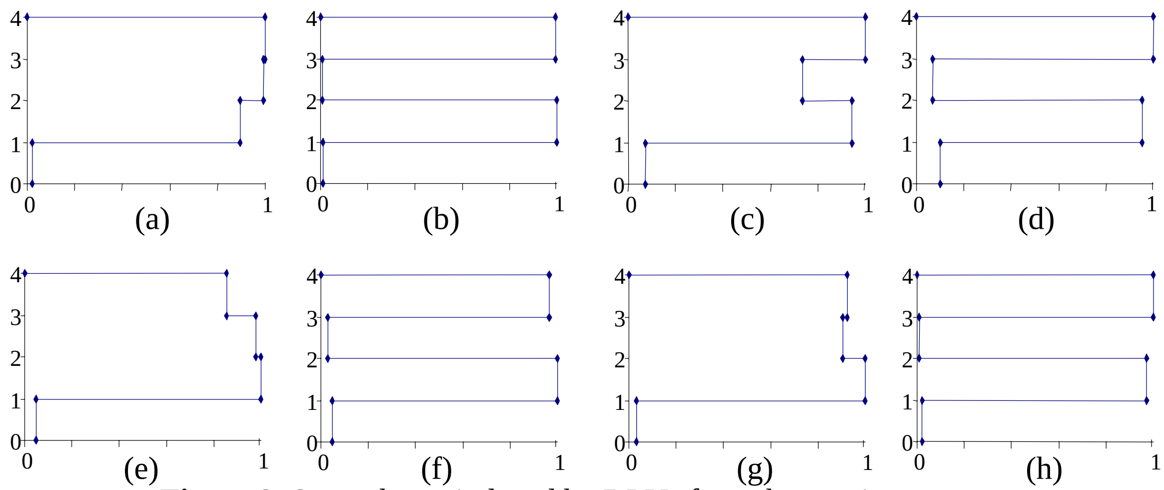

Figure 2 Story shears induced by DLVs for unknown input cases:
(a) Case 1-Damage 1 (b) Case 2- Damage 2 (c) Case 2- Damage 1 (d) Case 2- Damage 2
(e) Case3x-Damage 1 (f) Case3x-Damage 2 (g) Case3y-Damage 1 (h) Case3y-Damage 2

reduction in inter-story stiffness can not in this case be compared to an "exact result" since there is no exact inter-story stiffness (i.e, the ratio of shear to drift is dependent on the lateral load distribution). Notwithstanding, it is interesting to note that the procedure correctly identifies the inter-story stiffness of the first floor, which has a full rotational restraint at the base, as substantially larger than the value of the other stories.

References

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