

PERFORMANCE OF AN OBSERVER STATE-SPACE IDENTIFICATION IN THE PRESENCE OF MILD NONLINEARITIES

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ABSTRACT

The eigensystem realization algorithm with an observer Kalman Filter (ERA-OKID) operates under the assumption that the system is linear and time invariant. Most practical engineering structures, however, behave in a nonlinear fashion to a certain extent. The aim of the investigation reported in this paper is to examine how the system parameters identified by the ERA-OKID algorithm vary as nonlinearity is introduced in the system that generates the response data. The paper presents a review of the theoretical foundation of the ERA-OKID approach and offers numerical examples using two hysteretic structures. The first is a two-dimensional, four-story shear building and the second is a three-dimensional building with plan eccentricity, also with four-stories in elevation. In all cases, the excitation is taken as horizontal ground acceleration with bidirectionality considered in the three-dimensional case. The influence of the nonlinear behavior on the eigensolution and the potential for using the identified trends to characterize the nonlinear behavior are discussed.

1. INTRODUCTION

The development of active control strategies that can efficiently reduce the vulnerability of structures to accidental loads such as extreme winds and earthquake is recognized as an important objective structural engineering. Since many control algorithms rely on the availability of an accurate mathematical model of the structure, interest in the use of system identification techniques in Civil Engineering has increased sharply in the last decade sharply in the last decade. Significant attention has resulted also from the potential effectiveness of system identification techniques in the detection of extreme events, and for extracting information on the deterioration of structures over time.

A theoretical framework that has proved convenient and fruitful for the development of mathematical models from input/output data is the state-space approach. Among methods that operate entirely in time domain, the Observer/Kalman Filter Identification (OKID) algorithm has shown to be efficient and robust. A particular noteworthy feature of the algorithm is the introduction of

an observer that transforms the mathematical structure to one where the eigenvalues of the system matrix are zero for noiseless data or nearly zero when noise is present. A consequence of the modification introduced by the observer is that the non-zero length of the modified system's pulse response functions is drastically reduced when compared to those of the original system, with important gains in efficiency and robustness resulting. In effect, the OKID algorithm treats the output data as resulting from a modified input on a modified system. The matrices of a minimum realization for the actual system are then subsequently obtained for the modified system. The name Observer/Kalman derives from the fact that the gain of the introduced observer is that of a Kalman filter.

The OKID algorithm, as is the case in most of the system identification techniques commonly used in practice, operates under the assumption that the system being considered is linear and time invariant. In Civil Engineering applications, however, the assumption of linearity is hardly ever satisfied. Even in cases of relatively small excitation intensities, closing and opening of micro-cracks in concrete structures, yielding of regions with high residual stresses in steel structures, and the ever present interaction between the structural framework and non-structural elements, introduces nonlinearity in the response.

Current research efforts are directed towards the detection and identification of nonlinearity in structures. The review articles by Billings (1980), Tomlinson (1986), Natke, et al. (1998) Imregun (1998) reveal a survey of nonlinear system identification algorithms. However, because of their specialized nature and limited applicability, there seems to be some consensus that selection of a particular algorithm depends on the objectives of the analysis. The fundamental objective of this study is to examine the effect of mild nonlinearities in the response of the system on the system parameters identified by ERA-OKID algorithm. In this investigation the hysteretic behavior is based on Bouc-Wen model, which is mathematically convenient and can provide a good approximation to the conditions found in practice. The type of nonlinearity introduced is of hysteretic type.

2. Eigensystem Realization Algorithm with Observer Kalman Filter

The eigensystem realization algorithm (ERA) was first proposed by Juang and Pappa (1985) for modal parameter identification and model reduction of linear dynamical systems and was later refined and ERA with data correlations (ERA/DC) was formulated to handle the effects of noise and nonlinearities (Juang et al., 1988).

This technique, operating on pulse response functions, also known as Markov Parameters, produces an input/output mapping having the smallest state vector dimension that is compatible with a given accuracy. This mapping is known as a realization and has the form

$$\dot{x} = Ax + Bu \quad (1a)$$

$$y = Cx + Du \quad (1b)$$

where x = state vector, y = output vector, and the matrices A, B, C and D are the result of the realization. Since the same input/output mapping of eq.(1) is also given by;

$$\dot{z} = T^{-1}ATz + T^{-1}Bu \quad (2a)$$

$$y = CTz + Du \quad (2b)$$

it is evident that the matrices that define the realization are not unique (except for D which is independent of the non-singular transformation matrix T). Note, however, that since the system matrices of any two realizations are related by a similarity transformation, the eigenvalues are preserved.

In order to carry out the system realization with the extracted impulse response data, the discrete counterpart of the continuous state-space model can be expressed as

$$x(k+1) = A_1x(k) + B_1u(k) \quad (3a)$$

$$y(k) = Cx(k) + Du(k) \quad (3b)$$

For a system with r input and m measurement vectors, the system response, $y_j(k)$ at time step k due to unit impulse u_j can be written as

$$Y(k) = [y_1(k) \quad y_2(k) \quad \dots \quad y_r(k)], \quad k=1,2,\dots \quad (4)$$

and form the $ms \times rs$ Hankel matrix

$$H(k-1) = \begin{bmatrix} Y(k) & Y(k+1) & \dots & Y(k+s-1) \\ Y(k+1) & Y(k+2) & \dots & Y(k+s) \\ \vdots & \vdots & \ddots & \vdots \\ Y(k+s-1) & Y(k+s) & \dots & Y(k+2(s-1)) \end{bmatrix} \quad (5)$$

where s is an integer that determines the size of the matrix

By definition, the submatrices $Y(k)$ correspond to the system Markov parameters and can be expressed as

$$Y(0) = D \quad (6a)$$

$$Y(k) = CA^{k-1}B \quad k=1,2,\dots \quad (6b)$$

The basic formulation of ERA starts with the factorization of the Hankel matrix using the singular value decomposition,

$$H(0) = U S V^T \quad (7a)$$

$$H(1) = U S^{1/2} A S^{1/2} V^T \quad (7b)$$

Thus, the following triplet is a minimum realization:

$$\hat{A} = S^{-1/2} U_1^T H(1) V_1 S^{-1/2} \quad (8a)$$

$$\hat{B} = S^{1/2} V_1^T E_r \quad (8b)$$

$$\hat{C} = E_m^T U_1 S^{1/2} \quad (8c)$$

where E_m^T is $[I_m \ O_m \ \dots \ O_m]$ and E_r^T is $[I_r \ O_r \ \dots \ O_r]$ and O_i is a null matrix of, I_i is an identity matrix of order i .

The basic formulation of the ERA requires the system's Markov parameters. An accurate identification of the Markov parameters is vital for accurate system realization. Identification of the system Markov Parameters has traditionally been carried out by Discrete Inverse Fourier Transformation (IDFT) of Frequency Response Functions (FRF). The approach used here, however, solves for the Markov Parameters directly in the time domain. The approach avoids the well-known difficulties associated with time-domain deconvolution by the introduction of an observer. The observer, when appropriately selected, leads to a state-space representation where the output is mapped to a modified input by a system whose pulse response functions decay much faster than those of the original system. As one anticipates, the Markov Parameters of the original system can be recovered from the observer gain and the Markov Parameters of the Observer Model.

The State-Space Observer Model is readily obtained from eqs.(1). Specifically, adding and subtracting Gy to eq.(1a) and defining

$$v = \begin{bmatrix} u \\ y \end{bmatrix} \quad (9)$$

one gets

$$\dot{x} = \bar{A}x + \bar{B}v \quad (10a)$$

$$y = Cx + Du \quad (10b)$$

where;

$$\bar{A} = A + GC \quad (11)$$

and

$$\bar{B} = [B + GD \quad -G] \quad (12)$$

Provided the system is observable, the eigenvalues of the modified system matrix eq.(11) can be placed arbitrarily. One very attractive alternative is to select G such that all the eigenvalues of \bar{A} are zero. In this case the resulting system matrix is nilpotent and the Markov Parameters of the Observer Model become identically zero after a finite (typically small) number of time steps. Because of the close relationship between the gain G that leads to zero eigenvalues in \bar{A} (dead-beat observer) and the Kalman Filter, the foregoing approach is known as the Observer/Kalman Filter Identification (OKID) technique. In practice, the term is in fact generally used to refer to the complete process of identifying the pulse response functions followed by the generation of a minimum realization, typically using ERA/DC. A detailed presentation of the ERA/DC and the OKID procedures can be found in Juang (1994).

4. Numerical Examples

The first example is a four-story shear building with the mass and the initial stiffness corresponding to each floor and the system frequencies as shown in Figure 1. The structure has 5% damping in all modes. The input excitation is taken as horizontal ground motion. The restoring force relationship for the first floor is assumed to be of the Bouc-Wen (Wen, 1976) type with parameters that result in a smooth transition from elastic to plastic behavior. The restoring force vs the drift of the first floor is shown in Figure 2. Other floors are assumed to remain linearly elastic.

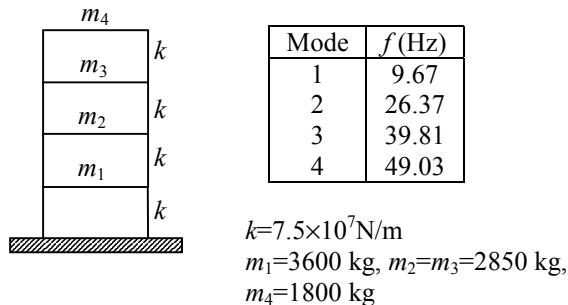


Figure 1 Structural model for Example 1

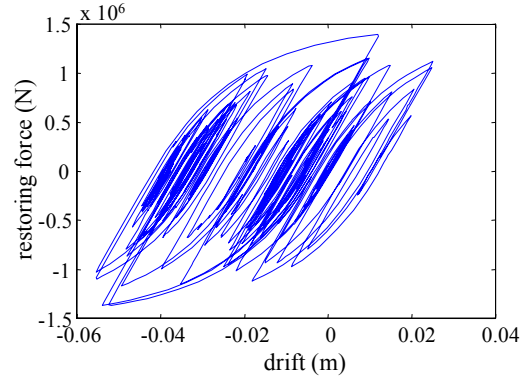


Figure 2 Restoring force vs. drift for the first floor

The input excitation is such that during the first 200 steps (0.8seconds) of the response, the system remains in its elastic range and after that the response becomes inelastic. Sensor noise is simulated by contaminating both the input and the analytically computed acceleration response for all four floors with white noise having an RMS equal to 5%. ERA-OKID procedure is carried out first by using the first 200 steps and the entire length of the response. The results are summarized in Table 1.

Table 1 Modal identification results

Mode	$Nsteps=200$		$Nsteps=2000$	
	ζ (%)	f (Hz)	ζ (%)	f (Hz)
4	5.65	49.13	5.44	49.08
3	5.25	39.67	5.35	39.40
2	5.09	26.29	6.82	25.59
1	5.23	9.65	12.31	9.33

Figure 3 displays the variation of the estimated damping ratios and the natural frequencies for the first mode as the number of steps used in ERA-OKID algorithm increases for both linear and nonlinear cases. This figure clearly indicates that while the estimates of frequency remains almost constant, the nonlinearity manifests itself with increased damping ratio estimates. It should also be mentioned that although the singular value decomposition of the Hankel matrix with 200 steps of the data indicates the order of the system as 8 (4 modes) during realization, as inelasticity progresses, it becomes harder to determine the order using the singular values and nonlinearity starts appearing as fictitious modes. In this example, once the order is determined in the first identification with 200 steps, the same value is used for the identification for different data sets.

After the identification is carried out and system realization is obtained, it is essential to test the accuracy of the identified modes in predicting the response time histories when subjected to different ground motions. The structure is excited with a recorded time history from the

Northridge (1994) earthquake (*Alhambra* station) and the acceleration response of the structure is simulated once again with the first floor having a hysteretic restoring force relationship. The prediction is based on the realization that is obtained using the entire length of the data. Figure 4 shows the acceleration time histories for the predicted and the actual accelerations of the fourth floor.

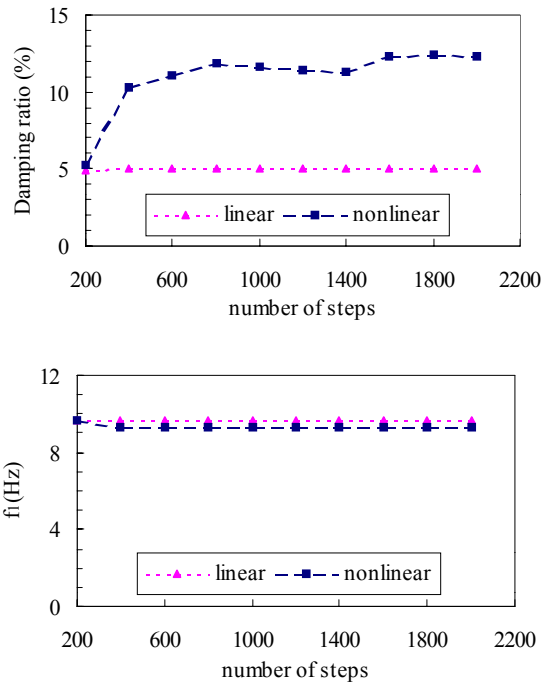


Figure 3: Number of steps used in ERA-OKID vs. the identification results for the 1st mode

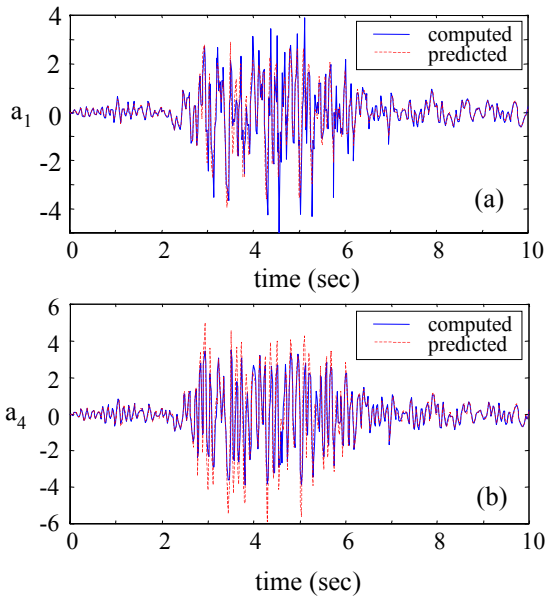


Figure 4: Computed vs predicted acceleration responses (a) 1st floor (b) 4th floor

The predicted response is very close almost identical to the actual response as long as the response remains linear. As the response changes from linear to nonlinear, the realization is no more capable of predicting the structural response.

In the second example, we have analyzed a three dimensional model of a four story shear building. The building has 2 equally spaced frames along the minor axis and 1 frame along the major axis. The translational stiffness of each floor are 7.5×10^7 N/m in the major direction and 2.9×10^7 N/m in the minor direction. The mass distribution is the same as the first example and the center of mass is eccentric on the second and fourth floors so that the torsional modes are also excited. The input excitation is horizontal ground motion in both x and y directions and the output measurements are the acceleration responses of each floor in both directions. The noise level for both the input and the output measurements are assumed to be 5%. The restoring force relationship for the resisting elements on the first floor along both the major and minor directions are assumed to be hysteretic as shown in Figure 5. The other floors are considered to remain linearly elastic during the entire response. Table 2 displays the identified values of the natural frequencies and the damping ratios with ERA-OKID approach. Except the last three modes, most vibration modes are picked up with good accuracy even under noisy conditions with only the first 200 steps of the response. The variation of the estimated values of damping ratios and the natural frequencies as the number of steps used in the identification values are shown in Figure 6. In the 3D case, similar to the first example, the frequency identification is found to be insensitive of the inelasticity and the effect is mainly reflected in an increase in the damping of the first mode

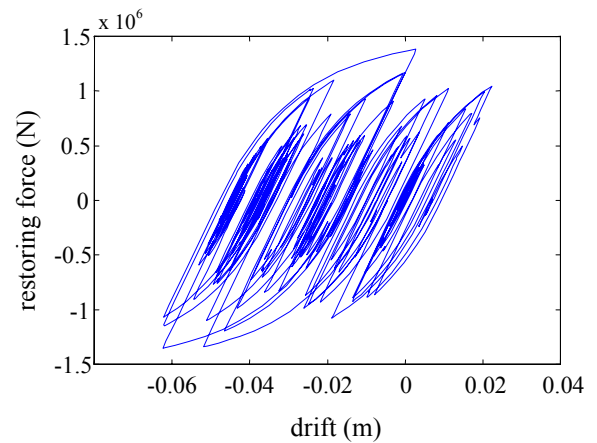


Figure 5: Example 2-Restoring force vs. drift for the element along major axis

Table 2 Modal identification results

Mode	Elastic (exact)		Nsteps=200		Nsteps=2000	
	ζ (%)	f (Hz)	ζ (%)	f (Hz)	ζ (%)	f (Hz)
1	5	8.26	6.12	8.10	16.77	8.00
2	5	9.28	6.02	9.30	13.61	9.04
3	5	10.74	5.93	10.75	12.00	10.52
4	5	22.53	5.16	22.45	6.94	21.75
5	5	25.26	4.47	25.08	6.65	24.52
6	5	28.41	5.63	28.37	8.11	27.93
7	5	35.07	5.68	35.16	7.89	35.15
8	5	38.67	6.22	38.06	6.17	37.90
9	5	42.73	5.11	42.93	5.59	41.91
10	5	45.63	7.51	49.66	8.18	48.12
11	5	47.67	-	-	-	-
12	5	55.57	-	-	-	-

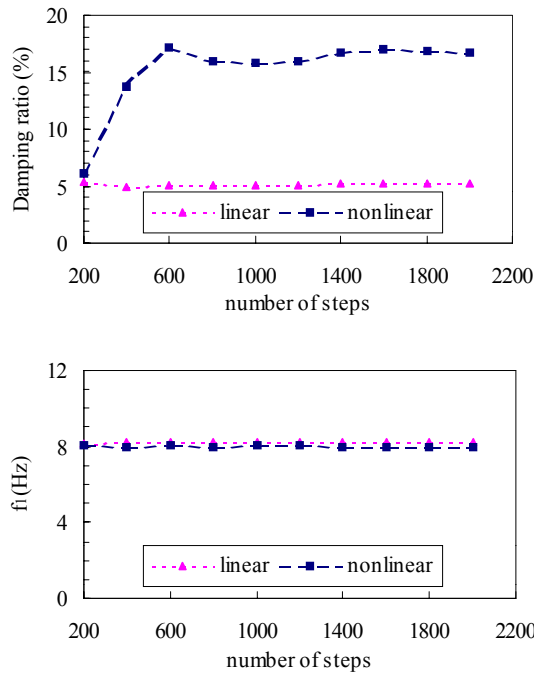


Figure 6: Number of steps vs. 1st mode

CONCLUDING REMARKS

In this study we have investigated the effectiveness of the ERA/OKID algorithm for applications to civil engineering structures with inelastic behavior. The hysteretic model investigated in this study offers a physically insightful model. The results of numerical simulations demonstrated that (a) nonlinearity in the system results in spurious modes (b) the estimation of natural frequencies of the system are quite insensitive to the inelasticity and the inelastic behavior is reflected in terms of increased damping ratios in the first vibration mode. In the context of damage detection technology this technique might provide a Level 1 damage identification

method in which one determines whether damage is present in the structure. It is also shown that the realized model mimics the behavior of the original system very well during the linear portion of the response when subjected to a different ground excitation. However as nonlinear action takes place, the realization is not capable of predicting the actual response of the structure. The realized model, therefore can not be used reliably to predict the response of the structure to a different ground motion excitation

REFERENCES

1. Billings, S. A. (1980) "Identification of non-linear systems: A survey," IEE Proc., vol.127, no.6, pp.272-285.
2. Imregun, M (1998) "A Survey of Nonlinear Analysis Tools for Structural Systems," The Shock and Vibration Digest, vol. 30, no.5, pp.363-369.
3. Juang, J.N. and Pappa, R.S. (1985) "An Eigensystem Realization Algorithm for Model Parameter Identification and Model Reduction," Journal of Guidance, Control, and Dynamics, vol.9, no.3, pp.294-303.
4. Juang, J.N., Cooper, J. E., and Wright, J. R. (1988) "An Eigensystem Realization Algorithm using Data Correlations (ERA/DC) for Model Parameter Identification," Control Theory and Advanced Technology, vol.4, no.1, pp.5-14.
5. Juang, J. N. (1994). Applied System Identification, Prentice-Hall, Englewood Cliffs, New Jersey.
6. Natke, H. G., Juang, J. N., and Gawronski, W. (1988) "A brief review on the Identification of Non-linear Mechanical Systems," Proc. IMAC 6, 1569-1574.
7. Tomlinson, G. R. (1986) "Detection, Identification and Quantification of Non-linearity-A Review," Proc. IMAC 4, pp.837-843.
8. Wen, Y. K. (1976) "Method for random vibration of hysteretic systems," Journal of the Engineering Mechanics Division, Proceedings of ASCE, vol.102.