

Load Vectors for Damage Localization

Dionisio Bernal, Member ASCE

Abstract: A technique to localize damage in structures that can be treated as linear in the pre and post-damage states is presented. Central to the approach is the computation of a set of vectors, designated as Damage Locating Vectors (DLVs) that have the property of inducing stress fields whose magnitude is zero in the damaged elements (small in the presence of truncations and approximations). The DLVs are associated with sensor coordinates and are computed systematically as the null space of the change in measured flexibility. The localization approach based on the use of DLVs is not structure type dependent and can be applied to single or multiple element damage scenarios. Knowledge about the system is restricted to that needed for a static analysis in the undamaged state, namely, the undamaged topology and, if the structure is indeterminate, the relative stiffness characteristics. Numerical simulations carried out with realistic levels of noise and modeling error illustrate the robustness of the technique.

INTRODUCTION

Research on vibration based damage identification has been expanding rapidly over the last decade. The situation most often considered is that where the system can be treated as linear in the pre and post damaged states, making damage tantamount to a shifting of values in a set of system parameters. In the context of this restricted definition damage characterization can be viewed as falling in the realm of linear model updating. A fundamental difficulty, however, lies in the fact that the inverse problem posed is typically ill-conditioned and, given the constraints imposed by the available data, generally non-unique (Udwadia 1985; Berman 1989; Beck and Katafygiotis 1998). Updating techniques arrive at the damaged system properties using constrained optimization algorithms that selects a particular solution from the set of possible ones; for example, strategies that use matrix perturbations of minimum rank or minimum norm have been widely used (Baruch and Bar Itzhack 1978; Kabe 1985). Although consistent with the modal data, the physical parameters obtained from these formulations may be unrelated to the actual values in the damaged system.

The likelihood of success in the damage characterization problem holds an inverse relationship to the size of the free parameter space that needs to be considered in the formulation. One way to arrive at a reduced set of candidate parameters for updating is by using information contained in the modal identification to restrict the portion of the domain where the damage is localized. Methods to localize damage without using a detailed model of the structure have traditionally been based on changes in mode shapes or mode shape derivatives or in changes in flexibility matrices assembled from the available modes (West 1984; Fox 1992; Salawu 1995; Pandey et.al. 1991; Stubbs and Kim 1996; Pandey and Biswas 1994,1995; Toksoy and Aktan 1994). In addition to these, methods that operate in the frequency domain using changes in transmissibility have been explored recently (Sampaio et.al. 2000). An examination shows, however, that most of the existing techniques are heuristic, conceived for particular types of structures, and few can operate with multiple damage scenarios and with an arbitrary number of sensors. An excellent review of the literature on the damage characterization problem up to 1996 can be found in Doebling et. al (1996).

A damage localization method based on changes in measured flexibility that is general and has a clearly tractable theoretical base is presented in this paper. The technique identifies the elements of the structure that are damaged as belonging to the set of elements whose internal forces under the action of a certain set of load vectors are zero. These vectors, which are designated here as Damage Locating Vectors, or DLVs, define a basis for the null space of the change in flexibility and can, therefore, be computed from the measured data without reference to a model of the structure. Because the stress fields used to locate damage are obtained with reference to a particular model of the structure, damage localization using the DLV technique is not immune to modeling error. The method, however, is generally insensitive to this

source of error because only the topology and the values of the relative stiffness parameters enter into the computation of the stress fields. As is shown in the body of the paper, the DLV technique is capable of considering single or multiple damage scenarios and can operate with a truncated modal basis and an arbitrary number of sensors.

As one gathers from the preceding discussion, the DLV approach is a technique to interrogate changes in measured flexibility with respect to the localization of damage. In this regard it is opportune to note that the computation of flexibility matrices from vibration data demands that one extract undamped, mass normalized modes from the measurements. Identification of undamped modes from a truncated and spatially incomplete basis of complex modes is a difficult problem for which an exact solution may not be feasible (Ibrahim and Sestieri 1995). Modal complexity, however, is typically small in lightly damped systems and in systems with well-separated frequencies (Gawronski 1998) and thus, for most conditions of practical interest, it is possible to obtain accurate estimates of the undamped modes by simple rotations. The need for the modes to be mass normalized, however, introduces a difficulty that may be significant in practice because it requires that the system identification be carried out for the case of deterministic input (Alvin and Park 1994). While the foregoing suggests that damage localization based on changes in flexibility can not be applied when ambient excitation sources are used, recent work by the author has shown that the limitation is not as severe as it may appear at first. In particular, while the change in flexibility can not be extracted from the data for stochastic input, it is often possible to extract a matrix whose null space is identical to that of the change in flexibility and this is all that is required to apply the DLV technique. The details of the approach will be presented in a forthcoming paper.

The remainder of this paper is organized as follows: the connection between the DLVs and the null space of the matrix given by the change in flexibility is presented first and is followed by a proof of the property of the stress fields induced by these vectors. The next section examines the relation between damage and the rank of the change in flexibility and considers the theoretical limitations of the spatial resolution associated with a given sensor set. The theoretical part of the paper concludes with development of a rational approach for selecting the thresholds that are needed to apply the technique when the modal information is imprecise and truncated. The guidelines developed in this section remove all subjective decisions and allow for an unsupervised application of the approach. Two numerical examples where the technique is applied to a truss having 44 bars and 9 sensors are presented. In the first example the flexibility matrices for the truss are derived from state-space system identification results and the localization is carried out for two different damage scenarios. The second example uses a truncated basis of analytically computed modes to assemble the flexibility matrices but examines 250 multiple damage cases to gain a statistical sense of performance.

THEORETICAL FORMULATION

Consider a system that can be treated as linear in the pre and post damage states, but which is otherwise arbitrary, having damaged and undamaged flexibility matrices at m sensor locations given by F_D and F_U respectively. To establish the theoretical foundation of the localization approach we consider first the ideal situation where the flexibility matrices are exact. Performance of the technique when these matrices reflect error in the identified modal parameters and truncation of the modal space is examined in a later section. Assume there are a number of load vectors, defined in sensor coordinates, which produce identical deformations at the sensors in the undamaged and the damaged states. If all the linearly independent vectors that satisfy this requirement are collected in a matrix L it is evident that one can write;

$$(F_D - F_U)L = DF \cdot L = 0 \quad (1)$$

The relationship in eq.1 may be satisfied in two ways, either $DF = 0$, or DF is rank deficient and L is a basis for the null space. The first possibility implies that there is either no damage or, as will become clear as the derivation in this section progresses, that the damage is confined to a region of the structure where the stresses are zero for any loading in sensor coordinates. In practice, of course, one never computes DF as identically zero so the real issue is to distinguish DF matrices that derive from damage from those that are purely the byproduct of inevitable fluctuations. While the issue is important for low levels of damage we restrict our attention in the remainder of this paper to the case where there is damage and the associated $DF \neq 0$. From a singular value decomposition (SVD) one gets;

$$DF = [U] \begin{bmatrix} s_r & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V}^T \\ L^T \end{bmatrix} \quad (2)$$

where $U \in \mathfrak{R}^{m \times m}$ contains bases for the column space and the left null space, $s_r \in \mathfrak{R}^{r \times r}$ is diagonal and contains the non-zero singular values, $\tilde{V} \in \mathfrak{R}^{m \times r}$ is a basis for the row space and $L \in \mathfrak{R}^{m \times (m-r)}$ is a basis for the null space. The property of the vectors in L that is relevant for damage localization is the fact that these vectors, when treated as loads on the system, lead to stress fields that are zero over the damaged elements. The basic idea, therefore, is that the intersection of the null stress regions corresponding to the load distributions defined by the null space of DF can be used to localize the damage. The column vectors in L are designated here as damage locating vectors or DLVs.

Depending on the number and location of the sensors, the intersection of the null stress regions may or may not contain elements that are not damaged in addition to the damaged ones. Elements that are undamaged but which can not be discriminated from the damaged ones by changes in flexibility (for a given set of sensors) are denoted as inseparable. While it is always possible to specify a sensor arrangement that eliminates the possibility of inseparable elements for ideal conditions, when the modal information is approximate and incomplete it is necessary to operate with finite thresholds and perfect spatial discrimination (at the element level) can not be guaranteed. A discussion on the question of inseparability is helpful for clarifying some features of the DLV technique and, for this reason, the matter is taken up in a later section.

Proof of the Damage Locating Property of the DLVs

Consider a load vector defined in sensor coordinates acting on a system. If this vector leads to zero stresses in certain elements, and the properties of these elements change while the loads remain constant, the state of stress and strain in the system will not vary. It is evident, therefore, that a sufficient condition for a load vector to be in the null space of DF is that, when acting on the undamaged system, it leads to zero stresses over the region where the damage is located. It is not immediately apparent, however, given that the displacements at the unmeasured coordinates are undetermined, whether or not the zero stress condition is necessary for a vector to be in the span of L . The derivation that follows clarifies this matter.

Consider a linear structure that has been discretized using n DOF. For notational convenience we partition the vector of DOF as $y = [y_a \ y_b]^T$ with $a = 1, 2, 3 \dots m$ and $b = m+1, m+2 \dots n$, where m is the number of loaded coordinates. The total potential for the system, Φ , is given by;

$$\Phi_{(y_a \ y_b)} = U_{(y_a \ y_b)} - W_{(y_a)} \quad (3)$$

where U is the strain energy function and W is the potential of the loads. Assume now that the displacements at the loaded points for the equilibrium condition are known and are treated as constants. An inspection of eq.3 shows that for this condition the potential of the loads is a constant and Φ and U are reduced to functions of y_b . Invoking the principle of stationary potential energy one gets;

$$\delta \Phi_{(y_b)} = \delta U_{(y_b)} = 0 \quad (4)$$

which shows that the strain energy is stationary at equilibrium (with $y_a = \text{cst}$). Furthermore, recognizing that for stable equilibrium the total potential is a minimum, one concludes (by inspection of eq.3), that $U_{(y_b)}$ is also a minimum. For the purpose of the subsequent analysis it is convenient to summarize the previous results in a form that can be recognized as the theorem of minimum strain energy, namely:

Of all the admissible strain distributions that yield the correct displacement at the loaded coordinates, the strain field that satisfies equilibrium minimizes the strain energy.

Consider now a finite dimensional linear structure for which the flexibility has been synthesized before and after damage. The incremental flexibility has been computed and a certain null space L has been identified. We designate an arbitrary column vector from this null space as DLV_i . As fig.1 illustrates, the domain of the structure can be subdivided into an undamaged portion Ω_U and a number of regions where the stiffness properties have changed as a result of damage, which we collectively designate as Ω_D .

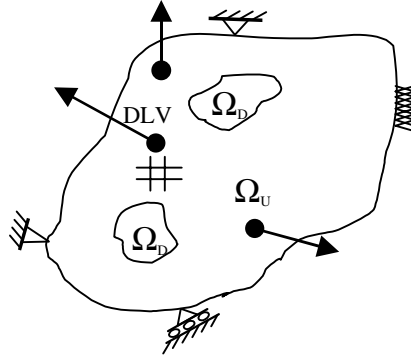


Fig.1 Schematic illustration of damaged and undamaged domains

The strain energy for the undamaged and the damaged states are given by;

$$U_U = \frac{1}{2} \left[\int_{\Omega_U} \boldsymbol{\varepsilon}_u^T \mathbf{E}_u \boldsymbol{\varepsilon}_u dV + \int_{\Omega_D} \boldsymbol{\varepsilon}_u^T \mathbf{E}_u \boldsymbol{\varepsilon}_u dV \right] = DLV_i^T y_a = q \quad (5)$$

$$U_D = \frac{1}{2} \left[\int_{\Omega_U} \boldsymbol{\varepsilon}_d^T \mathbf{E}_u \boldsymbol{\varepsilon}_d dV + \int_{\Omega_D} \boldsymbol{\varepsilon}_d^T \mathbf{E}_d \boldsymbol{\varepsilon}_d dV \right] = DLV_i^T y_a = q \quad (6)$$

where $\boldsymbol{\varepsilon}$ stands for the strain tensor, \mathbf{E} is the strain to stress mapping matrix and the subscripts u and d stand for undamaged and damaged states. Note that while $\boldsymbol{\varepsilon}$ is a function of position, we have opted to skip the subscripts x,y,z to keep the notation uncluttered. Needless to say, the material stiffness matrix, \mathbf{E} , may depend also on the x,y,z coordinates.

The next step is to obtain an expression that is either equal to, or is larger than U_D and we do this by invoking the minimum strain energy theorem. What is needed is simply to replace $\boldsymbol{\varepsilon}_d$ in eq.6 with any strain field that is geometrically admissible and leads to the correct deformation at the sensor locations. A

member of the set of admissible functions is the undamaged strain field ϵ_u . Substituting this strain field into eq.6 and expressing q by means of eq.5 one gets;

$$\int_{\Omega_U} \epsilon_u^T E_u \epsilon_u dV + \int_{\Omega_D} \epsilon_u^T E_d \epsilon_u dV \geq \int_{\Omega_U} \epsilon_u^T E_u \epsilon_u dV + \int_{\Omega_D} \epsilon_u^T E_u \epsilon_u dV \quad (7)$$

or;

$$\int_{\Omega_D} \epsilon_u^T E_d \epsilon_u dV \geq \int_{\Omega_D} \epsilon_u^T E_u \epsilon_u dV \quad (8)$$

The stiffness over the damaged region can be expressed in terms of the undamaged stiffness as;

$$E_d = \alpha_{(x,y,z)} E_u \quad (9)$$

where α is a scalar, and, since damage only reduces the stiffness, $0 \leq \alpha < 1$. Substituting eq.9 into eq.8 one gets;

$$\int_{\Omega_D} \alpha_{x,y,z} \epsilon_u^T E_u \epsilon_u dV \geq \int_{\Omega_D} \epsilon_u^T E_u \epsilon_u dV \quad (10)$$

which, given the fact that E_u is positive definite, can be satisfied only if the undamaged strain field (and as a consequence the stress field) is identically zero over the damaged region - thus completing the proof.

The fact that the proof depends on the premise that the changes from the undamaged to the damaged state are all stiffness reductions is worth restating. At first glance the requirement appears to have no practical significance, since damage always leads to reductions in stiffness. It is conceivable, however, that in addition to damage the changes between the two states may also include closing of previously open gaps or the consolidation of soil under supports which can increase the stiffness at some locations. When increases and reductions in stiffness take place simultaneously the damage bypass property of the stress fields induced by the DLVs can not be guaranteed. For the purpose of all subsequent discussion we assume that the damage scenario is defined by reductions in stiffness only.

Relationship between damage and the dimension of the null space of DF

It was shown in the preceding section that if a load vector leads to identical displacements at the sensors then the stress field that it creates in the undamaged system is null over the damaged region. The next important issue is to establish the conditions for the existence of a null space in DF . To clarify the relation between the rank of DF and the extent of damage consider again a structure that has been discretized using n DOF and has m sensors. Since the system has been assumed linear one can write;

$$RV = z \quad (11)$$

where R is an appropriately defined stress influence coefficient matrix, V is a vector of loads at the sensor locations and z is a vector containing the independent internal forces in all the elements. Consider now the scenario where a number of elements, say q_e , have been damaged and we wish to determine if there is a load vector V that will behave as a DLV, i.e., a vector that leads to zero stresses in all the damaged elements. To examine this question we assume (without loss in generality) that the damaged elements are listed first and write eq.11 in partitioned form as;

$$\begin{bmatrix} r \\ \bar{R} \end{bmatrix} \{V\} = \begin{Bmatrix} 0 \\ \bar{z} \end{Bmatrix} \quad (12)$$

Since eq.12 can be satisfied exactly only if r is rank deficient we conclude that the requirement for DF to have a null space is that the number of linearly independent rows in r be smaller than the number of sensors. We note at this juncture that the number of rows in r is not only a function of the number of damaged elements but also of the type of finite elements affected by the damage. In particular, if the number of deformation modes or, equivalently, the number of independent internal forces for a given element type is nb and there are qe -damaged elements, the number of rows in r is $\sum_{i=1}^{qe} nb_i$. In a truss, for example, $nb = 1$ and one concludes that the maximum number of damaged bars for which DF will have a null space (assuming these bars are independent for the sensor arrangement, and that there are no damaged bars that have identically zero stress for any $\{V\}$), is $(m-1)$, where m is the number of sensors.

Combining the previous analysis with the information contained in the proof on the property of the DLV vectors one concludes that the subspaces $N(r)$ and $N(DF)$, where $N(\cdot)$ stands for null space, are identical. The distinction of practical significance, however, lies in the fact that while $N(r)$ can only be computed when the damage is known and a model is formulated, $N(DF)$, i.e., the DLVs, can be computed from flexibility matrices synthesized from the measured data without any knowledge about the damage and without reference to any mathematical model of the system.

Inseparable Elements

From the partitions in eq.12 one gets;

$$V = N(r)\beta \quad (13a)$$

and

$$\bar{z} = \bar{R}V \quad (13b)$$

where β is an arbitrary vector of appropriate size. Combining eqs.13a and eq.13b one obtains;

$$\bar{z} = \bar{R}N(r)\beta \quad (14)$$

where \bar{z} is clearly the vector of stresses in the undamaged elements. An inspection of eq.14 shows that elements associated with any zero rows in the matrix $\bar{R}N(r)$ will have null stresses when the DLVs are applied to the structure and can not, therefore, be separated from the damaged elements by inspection of the stress fields. More generally, any element where the associated rows in \bar{R} can be moved to r without changing the rank of r can not be separated from the truly damaged set and are, therefore, inseparable. One concludes that under ideal conditions the DLV technique provides a set of potentially damaged elements that contains those that are truly damaged plus the inseparable ones. When working with imprecise and truncated modal data, however, finite thresholds become necessary and, as a consequence, the potentially damaged set is often larger than the theoretical minimum. Needless to say, the technique can not provide information regarding any portion of the system that is statically unobservable, i.e., a portion where the magnitude of the stress field is identically zero for any load vector defined in sensor coordinates.

Implementation of the DLV Technique.

The Normalized Stress Index - nsi

To discriminate between large and small stresses it is convenient to reduce the independent internal stresses in every element to a single value that we denote as the characterizing stress, σ ; where stress is used in a generalized sense to mean either an actual stress or a stress resultant. The characterizing stress is taken to be positive and is defined in such a way that the strain energy per unit length (or per unit area or volume, in 2D or 3D elements) is proportional to the square of its value. It is worth noting, for clarity, that elements with equal values of σ do not have equal values of mean strain energy (since the magnitude and distribution of the elemental stiffness also enters into the computations).

For a truss bar, for example, σ can be taken as the absolute value of the bar force. For a planar beam element there are two independent moments and, if the member is prismatic, σ can be taken as $(m_i^2 + m_j^2 + m_i m_j)^{0.5}$ where m_i and m_j are the two end moments. The normalized stress index nsi is defined as the characterizing stress in a given element normalized by the largest characterizing stress over all the elements of its kind.

$$nsi_i = \frac{\sigma_i}{\sigma_i|_{\max}} \quad (15)$$

Its worth noting that since σ is used in the damage localization only in the form of the ratio in eq.15 one can usually depart from the requirement that σ^2 be strictly proportionality to mean strain energy without introducing undue error. In the case of beam elements, for example, a definition of σ as the average of the absolute value of the end moments has been found to work just as well as the expression given previously.

The DLV Vector Set

The selection of a threshold to separate significant singular values from those that can be considered as defining the null space is ubiquitous in system realization theory (Akaike 1968; Rissanen 1978). The information from this knowledge base, however, does not appear to be directly applicable to the selection of a set of DLVs because the singular vectors that contain localization information often extent to singular values that need not be negligibly small. What is needed to select DLVs non-subjectively in a noisy environment is to identify an index that can be computed without knowledge of the damage but which is correlated to the expected maximum value of nsi over the damaged domain. The derivation of one such index is presented next.

We begin by expressing DF in terms of its singular value factorization, namely;

$$DF = U S V^T \quad (16)$$

where U and V are orthogonal, S is diagonal and contains the singular values and all three matrices $\in \mathfrak{R}^{m \times m}$. Pre and post multiplying eq.16 by a vector from V , say V_i one gets;

$$V_i^T DF V_i = V_i^T F_D V_i - V_i^T F_U V_i = s_i \quad (17)$$

which shows that the singular values of DF can be interpreted as the difference in the external work done by the associated singular vector when treated as a load on the damaged and undamaged states of the system. Assume now that the characterizing stress resultants induced by V_i in the undamaged and the damaged states are sufficiently close to be taken as equal (the assumption, of course, being exactly

satisfied if the system is statically determinate). Under this assumption the difference in work derives exclusively from changes in the strain energy in the damaged region and one can write;

$$s_i = \sum_{\Omega_D} \alpha_j \sigma_j^2 \quad (18)$$

where the subscript j refers to the particular element and the α 's are constants that depend on the size and the extent of the damage on each element but not on i . Multiplying the vector V_i by a constant, c_i , such that largest σ over the *full domain* is equal to unity one gets;

$$c_i^2 s_i = \sum_{\Omega_D} \alpha_j n s_i^2 \quad (19)$$

which can also be written as;

$$c_i^2 s_i = n s_i^2 \Big|_i \xi_i \sum_{\Omega_D} \alpha_j \quad (20)$$

where $n s_i^2 \Big|_i$ is the largest $n s_i$ in the damaged region and $0 < \xi_i \leq 1$. Assume now that the vector that leads to the largest value of eq.20 is that where $i=q$. Normalizing eq.20 with respect to its value for $i = q$, taking a square root on both sides and recognizing that the summations cancel out one gets;

$$n s_i^2 \Big|_i = \rho_i s v n_i \quad (21)$$

where;

$$s v n_i = \sqrt{\frac{s_i c_i^2}{s_q c_q^2}} \quad (22)$$

$$\rho_i = n s_i^2 \Big|_q \sqrt{\frac{\xi_q}{\xi_i}} \quad (23)$$

Eq.21 shows that estimation of the largest $n s_i$ over the damaged region is equivalent to the selection of a value for ρ (since the $s v n$ index can be computed without knowledge of the damage). While an upper bound for ρ can not be ascertained, we do know that $n s_i^2 \Big|_q \leq 1$ and the definition of ξ suggests that the expected value of the term in the square root of eq.23 may be taken as one. Estimates of $n s_i^2 \Big|_q$ based on $\rho = 1$, therefore, should prove conservative most of the time.

By choosing a value for $n s_i^2 \Big|_q$ that is not to be exceeded one obtains a cut on $s v n$ and therefore a set of vectors that qualify as DLVs. If the flexibility matrices are very accurate and the modeling error in the stress computations is small one expects optimum performance at very low values of $n s_i^2 \Big|_q$. In practice, of course, information on accuracy is not explicitly available so the cutoff for $s v n$ is best established to promote robustness. A value of 0.2 has been found to operate well for a wide range of conditions and is recommended. In summary, the DLV vectors may be taken as those that satisfy;

$$s v n \leq 0.20 \quad (24)$$

In the next section we combine the information from multiple DLVs in a way that introduces additional robustness into the technique.

Selection of the Set of Potentially Damaged Elements

The procedure is simple and can be succinctly described as follows: the potentially damaged set can be taken as those elements having $WSI \leq 1$, where;

$$WSI = \frac{\sum_{i=1}^{ndlv} \overline{nsi}_i}{ndlv} \quad (25)$$

where,

$$\overline{svn}_i = \max(svn_i, 0.015) \quad (26)$$

In eq.25 $ndlv$ is the number of DLV vectors and $\{nsi\}_i$ is the vector of nsi values for the i th DLV. Note that WSI is a weighted-average of the nsi values for each of the DLV vectors, with the weights being the reciprocal of the svn indices. The limit of 0.015 is introduced to prevent excessively large weights when svn is very small. Justification for taking the potentially damaged elements as those having $WSI \leq 1$ follows from eq.21 and the $\rho = 1$ selection.

A few closing comments are appropriate. We note that the potentially damaged set has not been defined using the intersection of the sets for each of the individual DLVs. The reason for this decision can be appreciated by noting that, in an intersection based approach, truly damaged elements are missed whenever the actual ρ for any DLV happens to be greater than the value used to make eq.21 quantitatively useful. Because ρ displays significant variability, the probability of missing a damaged element in an intersection approach increases with the number of DLV vectors and this is clearly an undesirable feature. The robustness introduced by the average of eq.25 is significant when there are multiple DLVs and extensive results show that this is realized with very little penalty on the size of the identified set. It is realized that elements where the numerator of eq.25 is large, in *any* DLV (say >2.5), could be eliminated from the potentially damaged set independently of their final WSI index. A provision along these lines, however, was tested in the simulation studies and proved to be an unnecessary refinement. We note, finally, that because of the nature of the computations the WSI cut of 1.0 should not be treated as a rigid boundary. In a case, for example, where $WSI = \{0.3 \ 0.7 \ 1.1 \ 2.3 \ \dots\}$ one should take the set of potentially damaged elements as the first 3 elements.

Performance of the DLV Technique

Example #1.

The structure selected is a planar truss with 44 bars and a total of 39 DOF. All the bars are made of steel (with $E = 200$ GPa) and are taken to have an area of 64.5 cm^2 . As one appreciates from a cursory inspection of Fig.2, the system is statically indeterminate both externally and internally. The mass matrix is taken to be diagonal with a value of $1.75 \cdot 10^5 \text{ Kg}$ at each coordinate. Nine sensors recording motion in the vertical direction are located at each of the unsupported joints of the lower chord. The excitation used to generate the vibration data is random white noise applied in the vertical direction at nodes 5 and 7 and the output is taken to be acceleration data. Sensor noise is contemplated in the excitation and the computed response. The output noise is prescribed to have a root-mean-square (RMS) equal to 10 % of the RMS of the response measured on the sensor located at joint 2 and the input noise RMS is 5% of the excitation level. Modeling error is simulated by computing the stresses from the identified DLV vectors using a model where the bar areas are the true values multiplied by a random scalar that is uniformly distributed between [0.95 and 1.05]. Viscous dissipation is included as 2% constant modal damping. Two damage scenarios are examined:

1. Bar 8-9 in the lower chord, and the diagonal 3-13 have their area reduced by 50%.
2. Bar 18-19 in the upper chord has its area reduced by 30%, the area of bar 6-7 in the lower chord is reduced by 60%, and the diagonal 8-20 fractures.

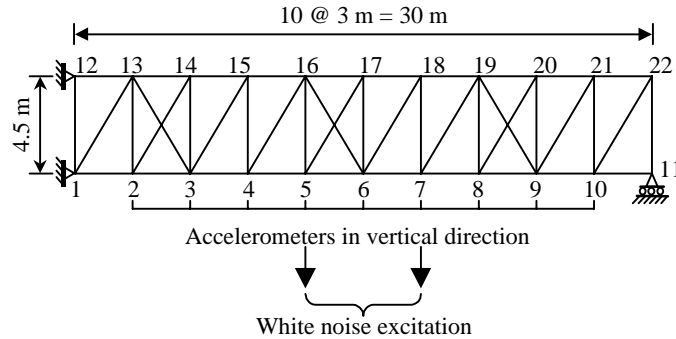


Fig.2 Structure Utilized in the Numerical Testing of the DLV technique.

In this example the ERA-OKID algorithm is used to perform the modal identification (Juang and Papa, 1985) and the modes are normalized with respect to mass, without introducing assumptions about the inertial characteristics, using a methodology described in Bernal (2000).

Exact Modal Periods

An appreciation of the frequency characteristics of the undamaged structure, as well as for the shifts introduced by the damage can be gained by inspecting Table 1 which lists the periods of the first 10 modes for the undamaged case and the two damaged scenarios. Note that the shift in periods are about 3.5% and 5% in the first mode and are very small in the other modes.

Table 1. Exact Periods for the First 10 Modes

Mode Number	Period (sec)		
	Undamaged	Damage-1	Damage-2
1	1.5520	1.6041	1.6310
2	0.8386	0.8433	0.8693
3	0.6486	0.6698	0.6523
4	0.4108	0.4133	0.4226
5	0.3054	0.3123	0.3269
6	0.2825	0.2864	0.2927
7	0.2551	0.2565	0.2692
8	0.2307	0.2314	0.2376
9	0.2105	0.2117	0.2135
10	0.2006	0.2057	0.2100

Modal Identification

The weighted-modal co-linearity index, $mpcw$, (Pappa, Elliot and Scenk 1993) was used to separate structural modes from noise modes. Application of the ERA-OKID algorithm to the simulated vibration data identified 24, 22 and 22 modes having $mpcw > 0.9$, in the undamaged case, and the two damaged scenarios, respectively. The $mpcw$ index measures the linear dependence of the real and the imaginary

parts of a mode and is unity when these are linearly dependent so the mode can be normalized to purely real. The index is useful because when the structure is lightly damped (or the frequencies are well separated) the true modes do not depart significantly from the modes of the undamped system, irrespective of the nature of the dissipation mechanism. In this example, of course, the damping is assumed classical so all modal complexity is a byproduct of approximations and noise. The periods of the first eight identified modes are summarized in Table 2. Comparison of the identified results with the exact values listed in Table 1 shows that the accuracy of the identification is excellent. The fact that the second mode from the exact solution doesn't show in the identification is due to the fact that this mode has essentially zero participating mass in the y-y direction.

Table 2. First 8 Periods Obtained from the ERA-OKID

Mode Number	Period (sec)		
	Undamaged	Damage-1	Damage-2
1	1.5527	1.6057	1.6337
2	0.6486	0.6697	0.6524
3	0.4107	0.4132	0.4226
4	0.3066	0.3134	0.3273
5	0.2781	0.2842	0.2960
6	0.2541	0.2564	0.2646
7	0.2304	0.2291	0.2402
8	0.2103	0.2115	0.2144

Flexibility Matrices

Fig.3 shows the percent error in the identified flexibility coefficients for the various cases. The errors are computed as the deviation between the computed flexibility coefficient and the exact value, normalized by the largest value in the associated column of the exact matrix. In this figure, the value of the index corresponds to the location in the matrix that is obtained by counting from the main diagonal downward, from column to column, sequentially. As can be seen from Fig.3, the identified flexibility is quite accurate, with the RMS of the error in flexibility coefficients being less than 1.5%.

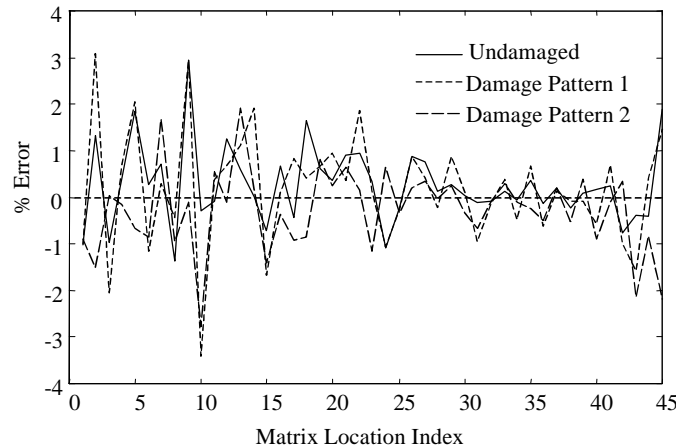


Fig.3 Error in the Coefficients of the Identified Flexibility Matrices

Computation of DLVs

The values of the *svn* indices are shown in Table 3 for the two damaged cases considered (listed in the order of the singular values of DF). Following the guideline in eq.25 one gets two DLVs in each case. The DLV vectors are listed in Table 4.

Table 3. *svn* indices

Damage Case 1	Damage Case 2
0.5037	0.6392
0.5458	1.0000
0.6674	0.9188
1.0000	0.3387
0.3256	0.5328
0.5055	0.3516
0.2872	0.1124*
0.1379*	0.3157
0.0361*	0.0826*

*DLVs according to eq.24.

Table 4. DLV for the Two Damage Cases Considered

Load at Joint	Damage Case 1		Damage Case 2	
	DLV ₈	DLV ₉	DLV ₇	DLV ₉
2	-0.5096	0.6457	0.4363	0.4811
3	-0.0084	0.0155	0.3867	-0.1193
4	0.1903	0.1218	0.4684	0.1386
5	-0.0836	-0.5044	0.1717	-0.6916
6	0.2256	0.2323	0.2251	-0.1825
7	-0.2986	-0.3660	0.0429	0.3824
8	-0.1402	0.2953	-0.3576	-0.0230
9	-0.0879	-0.0380	-0.3940	0.0470
10	0.7277	0.1927	-0.2749	0.2728

WSI Indices and Localization

Table 5 presents the *WSI* indices for the two damaged scenarios considered. As can be seen, the sets defined by $WSI < 1$ contain the bars that are actually damaged and include only two additional bars in damage case 1 and one in damage case 2. In a conservative interpretation of the results in Table 5 one would, of course, include the *WSI* values of 1.1 (bar 6-16 in damage case 1) and 1.14 (bar 10-21 in damage case 2) in the potentially damaged set also.

Table 5. *WSI* Values for the Two Damaged Cases

Damage Case 1		Damage Case 2	
Bar	<i>WSI</i>	Bar	<i>WSI</i>
8-9*	0.5552	6-7*	0.2670
3-13*	0.6606	18-19*	0.2670
15-16	0.9477	9-20	0.3856
3-4	0.9477	8-20*	0.4634
6-16	1.1055	10-21	1.1411
7-8	1.7651	8-19	1.2374

*damaged bars

** bars 10-11 and 1-12 are not included since their stress is identically zero for arbitrary loading.

Example #2 – Monte Carlo Simulation

We consider again the truss of fig.2 but in this case, to gain a statistical sense of performance, we obtain results for 250 damage cases defined by Monte Carlo sampling. The random variables are the number of

damaged bars, assumed to be either 1, 2 or 3, with equal probability, and the extent of the stiffness reduction in each bar, also assumed uniform with lower and upper limits of 20% and 60%. Because of the large number of simulations the flexibility matrices are not synthesized in this case from system identification results but are instead obtained analytically using a truncated set of modes. The number of modes used in all cases is 22, which is the smallest number identified for any condition in example#1. Modeling error is again contemplated by computing the stresses from the identified DLV vectors using a perturbed model. The model varies in each simulation and, as in example#1, is obtained by multiplying the area of each bar by a scalar that is uniformly distributed between [0.95 and 1.05]. The results are summarized in Fig.4. In particular, fig.4a shows the relative frequency with which damaged bars are missed and fig.4b depicts the relative frequency of the number of undamaged bars that are included in the identified set. Fig.4c shows also the undamaged bars in the set but in this case those that are theoretically inseparable are not included. An inspection of the numerical data shows that there were a total of 493 damaged bars and that in only one occasion was the *WSI* of a truly damaged bar larger than unity. In the single case missed *WSI* was 1.17 and the bar was the 3rd in the ranking according to *WSI* values so it is unlikely that a user would not have included in the potentially damaged set it in a supervised operation. The performance of the technique, regarding selection of a set that contains the truly damaged bars was, therefore, essentially perfect. As fig.4b shows, the number of bars that are not damaged but which are included in the potentially damaged set is satisfyingly small. Specifically, summing up all the undamaged bars in the sets and dividing by the number of simulations one obtains an average of 3.90 bars added, and, when the inseparable bars are removed, the average decreases to 1.54.

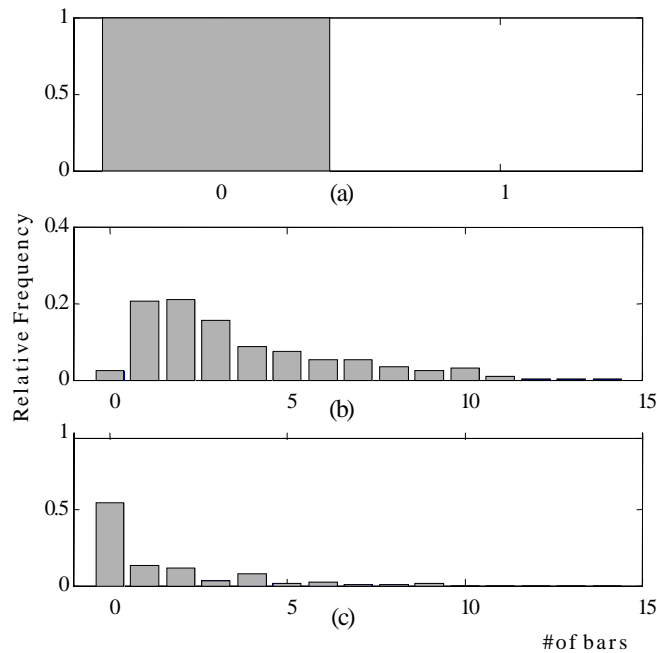


Fig.4 Histograms for 250 simulations of damage in the truss of fig.2;

a) missed bars b) undamaged bars in the identified set, c) same as (b) with inseparable bars removed.

Conclusions

Methods for damage localization have traditionally focused on finding *differences* between the undamaged and the damaged structure, i.e., differences between mode shapes, differences in deformed shapes due to an applied load, etc. The Damage Locating Vector technique introduced in this paper localizes damage using load distributions for which the static response of the structure is *the same* in the

undamaged and the damaged states and thus, in a certain sense, it represents the complement of the traditional strategy.

The DLV technique is conceptually simple and theoretically tractable. Indeed, for perfect data the performance of the technique can be examined analytically and simulations are found necessary only to gain an appreciation for the robustness of the method. Generality with respect to the type of structures that it applies to, its ability to treat single and multiple damaged element scenarios and the fact that the technique can operate with an arbitrary number of sensors without recourse to DOF expansion or reduction strategies are some of its attractive features. Finally, the fact that the DLV vectors are computed strictly from the measured data without reference to any mathematical model of the system is worth restating.

While the DLV approach has been introduced and discussed in the context of damage identification from vibration recordings, it is obvious that the approach is equally applicable when the flexibility matrices are obtained from static measurements. In fact, in the static case one anticipates fewer difficulties since the truncation that is ubiquitous in the dynamic situation is eliminated and only measurement noise remains as an error source for the flexibility matrices.

The numerical examples used to test the robustness of the technique incorporate most of the complications that are encountered in actual applications, i.e., noise in the measurements of input and output signals, multiple damaged members with damage of different severity, limited sensors, modal truncation and modeling error. As described in detail in the body of the paper, the DLV approach had no difficulty in identifying small sets containing the damaged elements in these cases (and many others not shown for brevity). The real test, of course, is whether or not the technique can operate successfully under field conditions. A final assessment on robustness awaits, therefore, experimental validation.

APPENDIX. REFERENCES

Akaike, H., (1968). "Information theory and an extension of the maximum likelihood principle." *2nd International Symposium and Information Theory*, B.N. Petrov and F.Caski, Eds., Akademiai Kiado, Budapest, pp.267-281.

Alvin, K.F. and Park, K. C., (1994). "Second-order structural identification procedure via state-space-based system identification", *AIAA Journal*, Vol.32, No.2, pp.397-406.

Baruch, M. and Bar Itzhack, I. Y., (1978). "Optimum weighted orthogonalization of measured modes." *AIAA Journal*, Vol.16, No.4, pp.346-351.

Beck, J. L., and Katafygiotis, L. S., (1998). "Updating models and their uncertainties I: Bayesian Statistical Framework." *Journal of Engineering Mechanics*, Vol.124, No.4, pp. 455-461.

Bernal, D., (2000). "Extracting flexibility matrices from state-space realizations." *COST F3 Conference*, Madrid, Spain, Vol.1, pp.127-135.

Berman, A., (1989). "Nonunique structural system identification." *Proceedings of the 7th International Modal Analysis Conference*, Society for Experimental Mechanics, Las Vegas, Nevada, pp.355-359.

Doebbling, S. C., Farrar, C. R., Prime, M. B. and Schevitz, D. W., (1996). *Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: a literature review*, Los Alamos National Laboratory, Los Alamos, New Mexico.

- Fox, C. H. J., (1992). "The location of defects in structures: A comparison of the use of natural frequency and mode shape data." *Proceedings of the 10th International Modal Analysis Conference*, San Diego, California, pp.522-528.
- Gawronski, W. K. (1998). *Dynamics and control of structures: a modal approach*, Mechanical engineering series, Springer-Verlag, New York.
- Juang, J., and Pappa R., (1985). "An eigensystem realization algorithm for modal parameter identification and model reduction." *Journal of Guidance Control and Dynamics*, Vol.8, No.5, pp.610-627.
- Ibrahim, S.R. and Sestieri, A., (1995). "Existence and normalization of complex modes in post experimental use in modal analysis." *Proceedings of the 13th International Modal Analysis Conference*, Nashville, Tennessee, pp.483-489.
- Kabe, A. M., (1985). "Stiffness matrix adjustment using mode data." *AIAA Journal*, Vol.23, No.9, pp.1431-1436.
- Pandey, A. K. and Biswas, M., (1994). "Damage detection in structures using changes in flexibility." *Journal of Sound and Vibration*, Vol. 169, No.1, pp.3-17.
- Pandey, A. K. and Biswas, M.,(1995). "Damage diagnosis of truss structures by estimation of flexibility change." *The International Journal of Analytical and Experimental Modal Analysis*, Vol.10, No.2, pp.104-117.
- Pandey, A. K., Biswas, M., and Samman, M. M., (1991). "Damage detection from changes in curvature mode shapes." *Journal of Sound and Vibration*, Vol.145, No.2, pp.321-332.
- Pappa, R. S., Elliot, K. B., and Schenk A., (1993). "Consistent mode indicator for the eigensystem realization algorithm." *Journal of Guidance and Control*, Vol.16. No.5, pp.852-858.
- Rissanen, J., (1978). "Modeling by shortest data description." *Automatica*, Vol.14, pp.465-471.
- Salawu, O. S., (1995). "Nondestructive assessment of structures using the integrity index method applied to a concrete highway bridge." *Insight*, Vol. 37, No.11, pp.875-878.
- Sampaio, R.P.C., Maia, N.M.M., Silva, J.M.M. and Ribero A.M.R., (2000). "On the use of transmissibility for damage detection and location." *COST F3 Conference*, Madrid, Spain, Vol.2, pp.363-376.
- Stubbs, N. and Kim, J. T., (1996). "Damage location in structures without baseline modal parameters." *AIAA Journal*, Vol. 34, No.8, pp.1644-1649.
- Toksoy, T. and Aktan, A. E., (1994). "Bridge-condition assessment by modal flexibility." *Experimental Mechanics*, Vol.34, pp.271-278.
- Udwadia, F.E, (1985). "Some uniqueness results related to soil and building structural identification." *SIAM Journal of Applied Mathematics*, Vol.45, pp.674-685.
- West, W. M., (1984). "Illustration of the use of modal assurance criterion to detect structural changes in an orbiter test specimen." *Proceedings of the Air Force Conference on Aircraft Structural Integrity*, pp.1-6.