Substitution Policies for a Hybrid System

Aybek Korugan and Surendra M. Gupta*
Laboratory for Responsible Manufacturing
334 SN, Department of MIME
Northeastern University
360 Huntington Avenue
Boston, MA 02115.

ABSTRACT

As a consequence of environmental necessities, reuse of products has recently become an important issue for production and planning. Many companies are involved in retrieving used products, where they repair, refurbish and upgrade the products in order to sell them for profit. However, the regulations for many markets do not allow manufacturers to sell remanufactured products under the same pretence as new products. Therefore, companies are forced to differentiate both the recovery and the sales activities for the remanufactured products from that of the new products. In this paper, we study the impact of this differentiation. We particularly look at the feasibility of substituting one version of the product with the other in order to satisfy the demand. In the first phase of the study, we try to find optimal switching functions for substitution decisions using a Markov decision process. In the second phase, we define several control policies and compare them with respect to the expected total cost function of the system.

Keywords: remanufacturing, hybrid systems, optimal switching policies

1. INTRODUCTION

As a consequence of environmental necessities, reuse of products has recently become an important issue for production and planning. Many companies are involved in retrieving used products, where they repair, refurbish and upgrade the products in order to sell them for profit. Such companies engage either only in remanufacturing of used products or produce both new and remanufactured goods. Companies that perform both activities are called *hybrid* companies.

In the literature, remanufactured parts and products are assumed to be restored to 'as good as new' condition. Models that study hybrid systems generally make this assumption[11, 13, 14, 15]. Remanufacturing is simply considered as another source to fulfill product demand. A state of the art review of remanufacturing models may be found in an extensive survey paper on product recovery by Gungor and Gupta[8] as well as in review papers by Guide *et al*.[7] and Fleischmann *et al*.[6].

Although some markets may accept remanufactured products as new products, the regulations for many others do not allow manufacturers to sell remanufactured products under the same pretence as new products. Therefore, a hybrid company is forced to differentiate both the recovery and the sales activities for the remanufactured products from that of the new products. However, since both new and remanufactured products have the same functionality they may substitute each other to fulfill a demand. In this paper, we look at the substitution cases where a penalty is associated with each substitution. Such problems are studied in the literature as procurement substitution, where a set of sub-products may fulfill each other's function in a production system[5]. Product substitution, in terms of quality levels, is encountered in inventory holding problems of perishable items. Adachi *et al.*[1] deal with products that deteriorate to three different quality levels over time and are consumed by a single demand, while Ishii and Nose[10] analyze a system with two different demands, one of which has priority over the other.

*Correspondence: e-mail: gupta@neu.edu; URL: http://www.coe.neu.edu/~smgupta/
Phone: (617)-373-4846; Fax: (617)-373-2921

In this study, we address the optimal switching problem for end-products capable of replacing each other in order to satisfy a demand. We define the problem in *Section* 2. Then in *Section* 3, we first give a stochastic dynamic programming model for the average total cost calculation. Then, we show that there exists a switching function that divides the feasible region. In *Section* 4, we introduce several switching functions. Finally, in *Section* 5 we conclude our work by comparing these policies.

2. PROBLEM DESCRIPTION

2.1 Notation

The following notation is used throughout the paper:

d_{rm}	The ratio between the costs associated with remanufactured and new items.
l_n	Demand rate for new products.
$\breve{l}_n$	Uniformized l_n
l_{rm}	Demand rate for remanufactured products.
$\breve{l}_{rm}$	Uniformized l_{rm}
m_n	New item production rate.
$\breve{m}_n$	Uniformized m_n
m_{rm}	Returned item remanufacturing rate.
$\breve{m}_{rm}$	Uniformized m_{rm}
p	Set of considered price policies.
c_n^l	Lost sales cost for a unit of unsatisfied demand.
h_n	Holding cost of a new item.
h_{rm}	Holding cost of a remanufactured item.
s_n	Sales price of a new item.
s_{rm}	Sales price of a remanufactured item.
x	State vector of the system.
x_n	Inventory holding level at the new item inventory.
x_{rm}	Inventory holding level at the remanufactured item inventory
Γ^p	Expected total cost of policy p .
$R_{i,k}^p(x_k)$	Revenue obtained from demand type *i* by selling a unit product of type *k* under policy p .
$V^p(x)$	Value function of policy p .

2.2 Problem Statement

We consider a production system that satisfies the demand for a product with both remanufactured and manufactured items. That is, the system accepts used product returns and brings them to 'as good as new' condition through disassembly,[3, 9] refurbishing, rework and upgrading activities to satisfy the demand. The system also fulfills the demand through its traditional manufacturing of new products. When the demand is not met, the system experiences lost sales. Through the remanufacturing (manufacturing) process, the products incur inventory holding costs.

The general assumption of remanufactured products being 'as good as new' enables the manufacturer to use them to satisfy the demand for new products without compromise. However, in reality, even though the remanufactured products are good enough to satisfy quality conditions set for new products, regulations in many countries do not allow manufacturers to sell remanufactured products or products manufactured with reconditioned parts under the same pretence as new products. This fact requires the remanufactured parts and products to be differentiated from the new parts and products. Hence, a hybrid production company must separate the remanufacturing and new product manufacturing activities. Obviously, this approach will result in products of two different types, viz, remanufactured (*rm*) and new (*n*) products. In our model, we assume this fact.

In order to underline the difference between the remanufactured and new products, we assume that the price s_{rm}, of remanufactured items, is a fraction, d_{rm} $(0 \leq d_{rm} \leq 1)$, of the price, s_n, of new items. Furthermore, we assume that the same relationship holds for inventory holding costs of remanufactured (h_{rm}) and manufactured (h_n) items. We also assume that there exists an infinite supply of returned items for the remanufacturing process and an infinite supply of raw material for the manufacturing process. The demand arrives according to a Poisson process with rate l_n (Figure 1). Here the demand is either satisfied with new items or remanufactured items that substitute new items for a predefined penalty. If the demand is not satisfied by either product type, it incurs a lost sales per unit cost of c_n'. For a hybrid production system described above, it is necessary to find a decision policy that determines when to substitute the new products with the remanufactured ones to satisfy the demand. The objective is to minimize the expected total cost of the system.

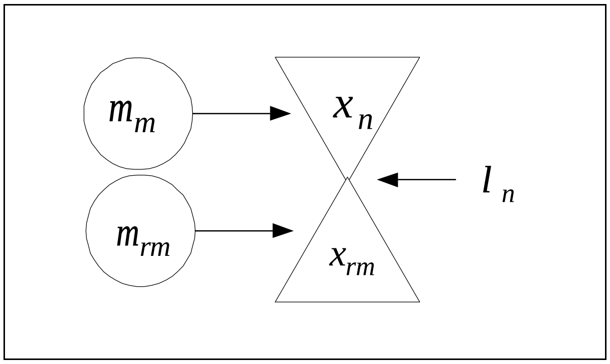

Figure 1 : The hybrid system

3. THE EXPECTED TOTAL COST

In our model, we define the additional cost for a substitution as a loss in revenue in terms of the missed opportunity. That is, when a remanufactured product is used to satisfy the demand for a new item, we miss the opportunity to satisfy the demand with a new item and hence to obtain a revenue s_n. Thus, when we satisfy the demand through a substitution we obtain a revenue of $s_n(2d_{rm}-1)$. Obviously, for a substitution to be worthwhile, $s_n(2d_{rm}-1) > c_n'$ should hold. Using this inequality, we find, $d_{rm} \geq (h_l+1)/2$ where $h_l = c_n'/s_n$, $0 \leq h_l \leq 1$. Thus, any $\{d_{rm}, h_l\}$ pair that satisfies this inequality ensures the substitution to create a profitable revenue[12].

As seen in Figure 1, the hybrid system of interest consists of two inventories. Let, $x = (x_{rm}, x_n)$ show the state of the system, with x_{rm} and x_n giving the number of items at the remanufactured and new item inventories respectively. Also, let e_i be the unit vector along the i th axis. Then, $x \to x + e_{rm}$ at the remanufacturing rate of m_{rm} and $x \to x + e_n$ at the manufacturing rate of m_n. Also, $x \to x - e_{rm}$ when a demand is satisfied by substitution and $x \to x - e_n$ when the demand is satisfied with a new product. As expressed earlier, all service times are exponentially distributed i.i.d. variables with the above given rates while the demand occurrences follow a Poisson distribution with mean l_n.

In order to find the expected total cost for the system, we define a holding cost function $h(x)$ with holding costs h_{rm} and h_n for the remanufactured and new product inventories, respectively. In addition, we subtract the revenue for each sold item. For this system, we define a value function[17],

$$\begin{aligned} V^p(x) = \Gamma^p + h(x) + \breve{m}_{rm} V(x+e_{rm}) + \breve{m}_n V(x+e_n) \\ + \breve{l}_n \min\{V(x-e_{rm}) - s_n(2d_{rm}-1),\, V(x-e_n) - s_n\}, \end{aligned} \tag{1}$$

with the expected total cost Γ^p and the uniformized transition rates $\breve{m}_{rm} + \breve{m}_m + l_n = 1$. Here, a decision matrix D associated with the state space $X = \{x \in Z : x_{rm}, x_n \geq 0\}$ gives the optimal decision rule for product substitutions at the convergence of the expected total cost (Figure 2).

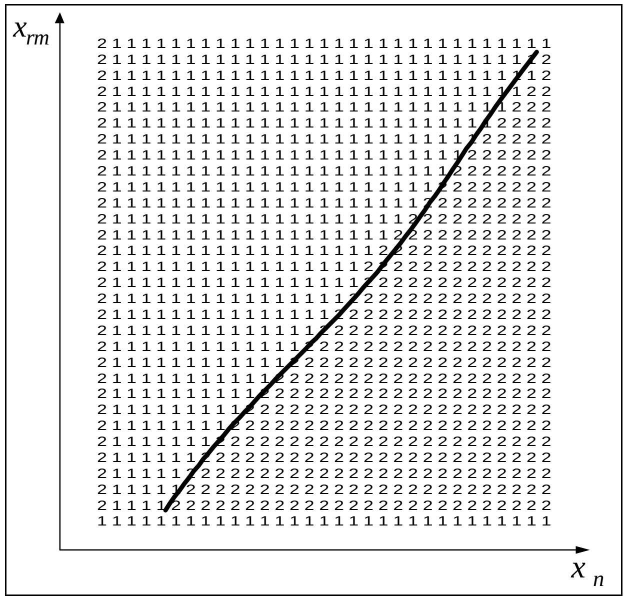

Figure 2: The Switching Function on the Decision Matrix *D*

4. THE SWITCHING FUNCTIONS

We conducted a set of experiments using equation (1) in order to determine the behavior of the decision matrix D under different circumstances. As a result of our observations we conclude that there exists a switching function $w(x)$ that divides the state space into two parts such that for all states where $x_{rm} \leq w(x_r)$ it is optimal to supply the demand with the new product, while it is preferable to meet the demand with the remanufactured product otherwise. Hence, the optimal control problem for product substitution is reduced to finding a switching function $w(x)$. In order to find near optimal control policies, we take two different approaches to the problem. The first approach consists of defining a linear switching function $w(x_n) = ax_n + b$ while the second approach involves the analysis of an equivalent fluid model [2].

The first approach consists of three policies, where we define a switching function for each of them. The first policy is based on a simple switching function $w_1(x_n) = x_n$ independent of cost parameters that divides the state space into two equal subspaces. A second policy is given by another switching function $w_2(x_n) = x_n + b$ where we determine b using the ratios between the holding costs and the revenue. Here, based on further observations we give $b = 1 - s_n / h_{rm}$, since the region for substitution decreases while $a_{rm} \rightarrow 1$ and the s_n / h_n ratio increases. Finally, we determine a slope $a = 0.7/1 - 2 l_n$ empirically and define a switching function as $w_3(x_n) = 0.7\, x_{rm} / 1 - 2 l_n + (1 - s_n / h_{rm})$.

For the second approach, we define an equivalent queueing system, where the demand process is regarded as service and both manufacturing and remanufacturing service rates are considered arrivals that compete to receive service (Figure 3). Here, a cm rule is used to determine the priorities[4, 2]. Thus, the decision rule is independent of the state $x \in X$ of the system. For the $h_{rm}\, m_{rm} < h_n\, m_n$ case we satisfy the demand only with new products while for the complimentary case we only serve remanufactured products.

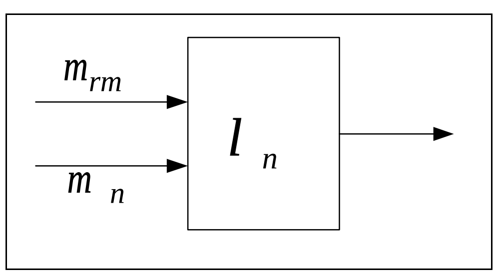

Figure 3:The equivalent Fluid Model

5. EXPERIMENTATION

We designed an experiment, $L_{27}(3)$ using orthogonal arrays[16]. We have 7 independent parameters; a full factorial design of three levels of each variable would require us to conduct $3^7 = 2187$ experiments. Obviously, it is both time consuming and impractical. By using the $L_{27}(3)$ orthogonal array, we were able to cover the entire experimental region with only 27 experiments while designing a three level experiment set for all parameters. The levels of the independent variables of the model are given in Table 1.

After constructing the orthogonal array using these values, we calculated the expected total costs for all four near optimal control policies given in *Section* 4 by guiding the predefined decision matrix D into the dynamic programming model. Then we compared the expected total costs these policies generated with the values obtained by the optimal control policy. For the sake of tractability, we have truncated the state space by an arbitrarily large value that enabled us a good approximation. In order to assure the wellness of the approximate values, we set the convergence margin of Γ^p to $e < 0.001$.

Table 1:Levels of the experimental design

Levels	h_n	m_{rm}	m_n	l_n	c_n^l	s_n	d_{rm}
1	4	1	0.6	0.4	6	20	0.5
2	10	1.4	1.2	0.6	25	50	0.7
3	20	1.8	2	0.8	70	100	0.9

The results are plotted in Figure 4. Here, the *x*-axis represents the experiments performed and the *y*-axis gives the expected total cost each substitution policy calculates. As can be seen, the switching functions $w_1(x_n)$ and $w_2(x_n)$ perform better than both the more complicated $w_3(x_n)$ and the fluid control in general. However, further experiments show that the effectiveness of these functions is limited to low traffic densities. That is, as $l_n/(m_n + m_{rm} + l_n) \to 1$, the accuracy of these functions dissipates.

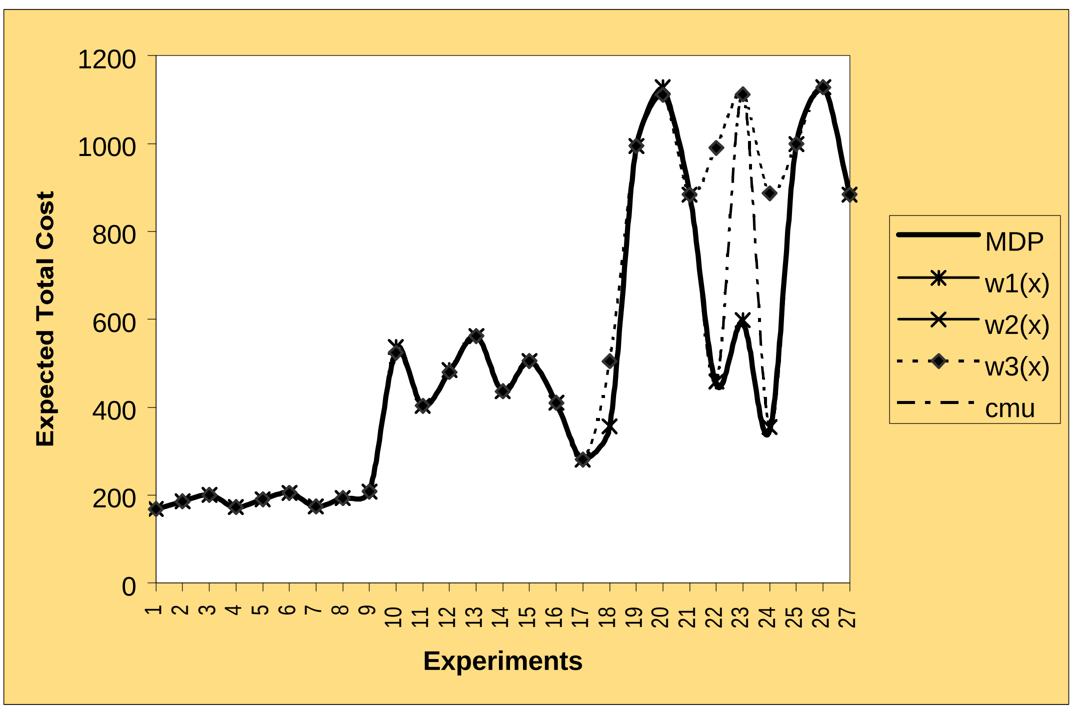

Figure 4: The Comparison of the Optimal Policy(MDP) with the linear ($w1(x), w2(x), w3(x)$) and fluid (cmu) Sub-optimal Policies

6. CONCLUSIONS

In this paper, we have studied a hybrid system that satisfies the demand for a certain type of product with either new items or remanufactured items. We have first observed that there exists a switching function that divides the state space into two parts and determines which type of product should satisfy the demand. Then, we introduced four different policies that generate near optimal results for the substitution control problem. We designed an experiment in order to test these four new policies against the optimal switching policy obtained through an infinite horizon stochastic dynamic programming model. We have observed that the state dependent switching policies perform better than the state independent policy defined by the equivalent fluid queue. We also found that the impact of the lost sales price is insignificant compared to the holding costs and revenues. We believe that the effect of failing to satisfy the demand on time can be easier on a model that permits backordering. Yet, in such a model the effect of the holding costs will still be a significant determinant for choosing the switching policy.

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