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FRICTION INDUCED TRANSVERSE VIBRATIONS OF AN AXIALLY ACCELERATING STRING

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ABSTRACT

The purpose of this study is to investigate the dynamic response of axially translating continua undergoing both the effect of friction and axial acceleration. The axially moving continuum is initially modeled as a string, neglecting its flexural stiffness; the response, with particular interest given to transverse vibrations and dynamic stability, is studied through numerical methods. A finite element method is employed to discretize the space domain and an implicit α–method is employed to integrate the resulting matrix equation in the time domain. Results are given through time history diagrams and stability considerations.

INTRODUCTION

Translating continua are encountered in numerous machinery such as power transmission belts, band saw blades, and processing systems such as magnetic tapes, thread lines, paper and photographic film.

Many researchers have addressed the axially moving continuum problems, but most of them have considered conservative systems without friction [1,2]. However, in many applications stationary frictional guides, which are potential sources of undesired vibrations and dynamic instability, exist. The response of a frictional, non-conservative system has been studied by Chen [3], and Cheng and Perkins [4], among others, who considered a string moving with constant axial velocity under the effect of a stationary dry friction load.

Accelerations and decelerations, as well as the undesirable fluctuations of the translational speed could have significant effects on the vibrational behavior of the translating systems. Miranker derived the governing equations of an accelerating string using energy methods [5]. Solutions of this and similar systems have been studied by Pakdemirli et al. [6], among others. To the best of our knowledge the combined effect of frictioned guides and acceleration has not been addressed.

In systems requiring frequent start-stop operations, such as high capacity data tape-recorders, simultaneous consideration of friction and acceleration is necessary. Here a generic model and a numerical method are presented to address such a system. The primary goal of this paper is to show the validity of numerical model by comparing the numerical results with that of reference [6].

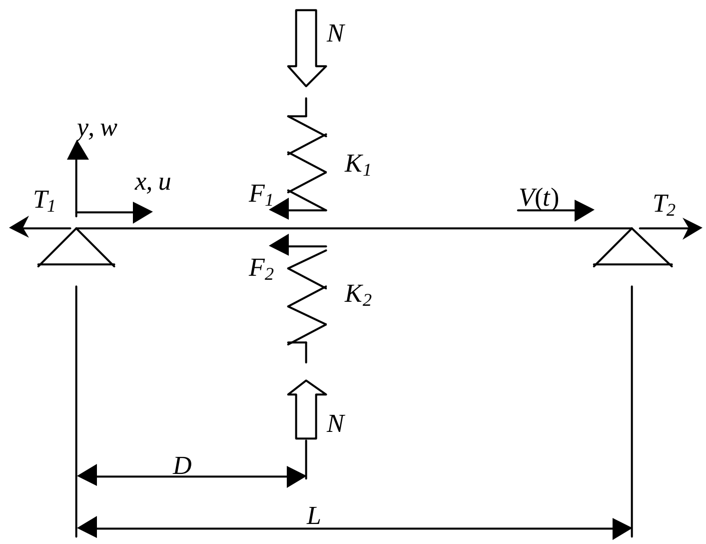

Figure 1. System modeled as an accelarating string subjected to stationary frictional loads F_1 and F_2.

EQUATION OF MOTION

A tensioned flexible string is translating with time dependent velocity $V(t)$ between two fixed supports at $x = 0$ and $x = L$. At a distance D from the left support there is a frictional guide resting on two springs. The surfaces of the guides are assumed to remain in contact with the string because of a static force N and the stiffnesses K_1 and K_2 of the guide supports. F_1 and F_2 are the top and the bottom friction forces due to the guide. The longitudinal and the transverse displacements are u and w, respectively.

In this paper the equation of motion is derived using the extended Hamilton's principle, assuming a time dependent translation velocity, a Lagrangian strain and a concentrated elastically supported frictioned guide. Equations have been derived both for the longitudinal and for the transverse vibration, ending up with a system of two coupled non-linear partial differential equations with time dependent coefficients:

$$r\left[u_{tt}+V_t(t)(1+u_x)+2V(t)u_{xt}+V^2u_{xx}\right]-EAu_{xx}=-T(t)_x \qquad (1a)$$
$$-(F_1+F_2)d(x-D)$$

$$r\left[w_{tt}+2V(t)w_{xt}+V^2(t)w_{xx}+V_t(t)w_x\right]-EA(u_xw_x)_x+\left[T(t)w_x\right]_x \qquad (1b)$$
$$+kwd(x-D)=-q$$

where the subscripts x and t indicate partial derivatives with respect to space and time respectively, w is the vertical displacement, r is the linear density, k the global stiffness of the guide supports, d the Dirac function, q the eventual distributed load, H the Heaviside function, T is the tension and E is the elastic modulus of the string. In general, the friction forces F_1 and F_2 depend on the vertical displacement w as follows,

$$F_1=m\left(N+k_1w(D,t)\right) \text{ and } F_2=m\left(N-k_2w(D,t)\right) \qquad (2)$$

In the time scale of the transverse motion, the longitudinal inertia term is neglected considering that the speed of propagation of longitudinal waves $(EA/r)^{1/2}$ greatly exceeds that of the transverse waves $(T/r)^{1/2}$. Thus the study reduces to the transverse displacement equation, that is now uncoupled from the longitudinal displacements. The resulting equation of motion is given below,

$$r\left[w_{tt}+2Vw_{xt}+\left\{V^2-T-(F_1+F_2)H(x-D)\right\}w_{xx}+V_tw_x\right] \qquad (3)$$
$$+kwd(x-D)=-q$$

This equation is solved subject to two boundary conditions. On the right-hand side support the vertical displacement of the string is constrained to zero. A harmonic motion is applied to the left-hand side support in order to excite its various natural frequencies. These conditions are expressed as follows,

$$w(0,t)=A_o\sin(w_ot), \qquad w(L,t)=0 \qquad (4)$$

where A_o and w_o are the amplitude and frequency of the support motion.

SOLUTION METHOD

Equations (2) - (4) are solved numerically. The total energy of the system is minimized as follows,

$$\delta\left(\int_{t_1}^{t_2}\left(E_K-E_P\right)dt\right)=0 \qquad (5)$$

where E_K and E_P are the kinetic and potential energies of the system, given by,

$$E_K=\frac{1}{2}r\int_0^L\left[\left(u_t+V(t)\left(1+u_x^2\right)\right)^2+\left(w_t+V(t)w_x\right)^2\right]dx \qquad (6)$$

$$E_P=\frac{1}{2}E\int_0^L\left[A\left(u_x^2+\frac{1}{4}w_x^4+u_xw_x^2\right)\right]dx-\int_0^L q(t)wdx- \qquad (7)$$
$$\int_0^L T(t)\left(u_x+\frac{1}{2}w_x^2\right)dx$$

Note that in this formulation the translation speed V, the tension T and distributed load q acting on the string are allowed to depend on time. Upon energy minimization, linear finite elements are used to discretize equation (5) in space, resulting in,

$$[M]\{a\}+[G]\{v\}+[K]\{w\}=\{q\} \qquad (8)$$

where $[M]$, $[G]$ and $[K]$ are the mass, gyroscopic inertia and stiffness matrices, and $\{a\}$, $\{v\}$, $\{w\}$ and $\{q\}$ are the nodal normal acceleration, normal velocity, normal displacement and load vectors, respectively. Resulting set of ordinary differential equations were integrated in the time domain using the alpha-method (Hilbert-Hughes-Taylor method [7]) with an implicit integration algorithm as described in [8].

A Matlab program has been developed to solve the final equation. In order to check its correctness, the results are compared to Cheng and Perkins (CP), where an analytical solution to a simplified problem has been presented [4]. In this case the translation velocity and the friction forces F_1 and F_2 are assumed to be constant. The following non-dimensional variables are defined by CP,

$$X = x/L,\ d = D/L,\ c^2 = T_2/r,\ g^2 = T_1/T_2,\ h = V/c,$$
$$T^* = tc/L,\ W = wL/c,\ K = 2KL/rc^2,\ W = w/L \qquad (9)$$

For the comparison the following values are used d = 0.2 and 0.5, h = 0.3, g = 0.5, K = 20. The excitation frequency W_o varies in the range 0 - 10 and A_o = 1. The external pressure q is zero. The time step Δt and the spatial node spacing Δx were chosen such that $c\Delta t/\Delta x = 1/\sqrt{3}$ for consistent mass matrices [7]. Relatively small amount of numerical damping was chosen by setting α = -0.05. The non-dimensional nodal spacing Δx was 1/100.

RESULTS

The transient response of the string is obtained numerically. The response envelope of the system is obtained by superposition of the string shape at different time steps. The excitation frequency corresponding to the maximum response envelope is determined.

Figure 2 shows comparison of the response envelopes for the case d = 0.2 and h = 0.3 predicted by CP and the numerical methods. There is a good agreement between the analytical and numerical results. The numerical method is capable of capturing the response envelopes, especially for the second, fourth and fifth mode shapes. In all cases the location of the bigger response envelope is predicted properly, however for the first and the third mode shapes, the relative amplitudes of the response envelope to the left and to the right of the frictional support (d = 0.2) are off, as compared to CP results.

***d* = 0.2**		***d* = 0.5**	
Cheng-Perkins	**Numerical**	**Cheng-Perkins**	**Numerical**
3.37	3.24	1.98	1.94
4.85	4.74	3.95	3.90
6.80	6.70	5.23	5.01
9.54	9.25	5.99	6.01
10.44	10.30	7.93	7.94

Table 1 Comparison of non-dimensional natural frequencies obtained by Cheng and Perkins [4] and the numerical method.

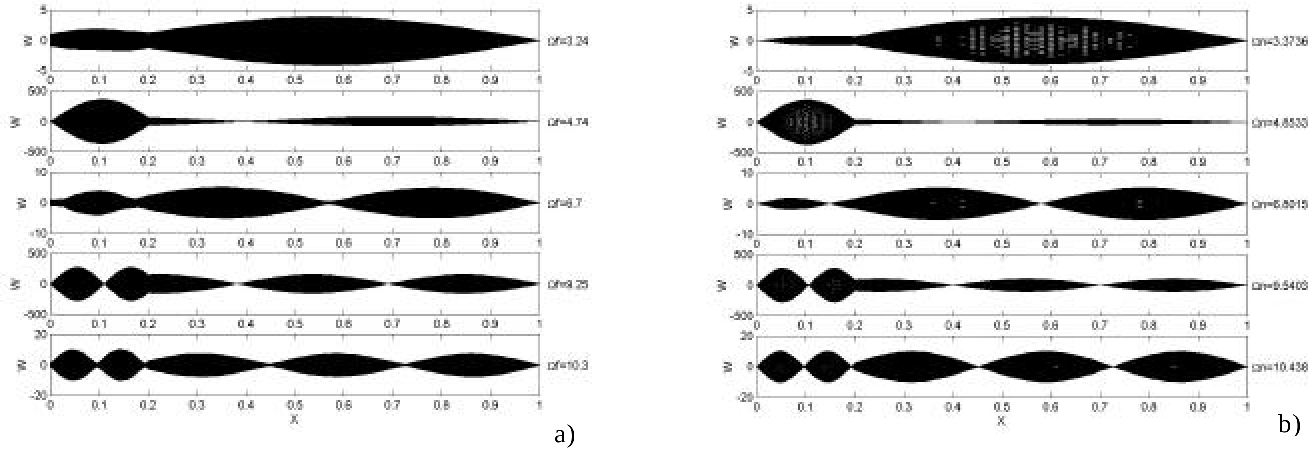

Figure 2. The first five natural frequencies and corresponding moc
Perkins' results, [4].

The maximum displacements W_{max} from Figure 2 are plotted as a function of the non-dimensional excitation frequency W_o in Figure 3. The first five natural frequencies of the system are reported in Table 1, which shows that the numerical method slightly underestimates the magnitudes of the natural frequencies.

SUMMARY AND CONCLUSION

A generic numerical method for analyzing the dynamics of a translating, tensioned string subjected to frictional point loads from top and the bottom is developed. The method is also capable of including the string acceleration. The effect of friction is tested with a known, relatively simpler analytical solution and good correlation is obtained. Future work will include the stability analysis of the numerical time integration method as applied to this problem, analysis of the dynamic stability of the accelerating system, as well as a system with acceleration and friction.

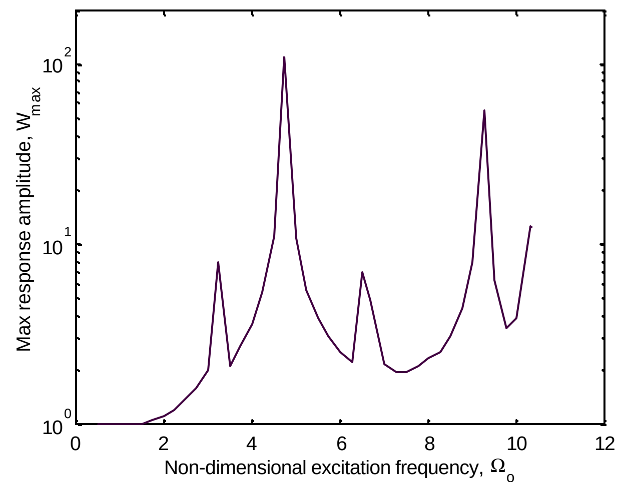

Figure 3. Numerically obtained non-dimensional frequency-amplitude diagram for d = 0.2 and γ = 0.5

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