TRACTION BETWEEN A WEB AND A SMOOTH ROLLER

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ABSTRACT

A general model is presented to predict the traction capability of an impermeable web over a smooth roller. The model considers the effects of the web and roller speeds, roller radius, combined roughness of the two surfaces, web tension and thickness, friction coefficient, and air lubrication. The change of tension ΔN due to mechanical slip between the roller and the web is calculated by a simultaneous solution of the in-plane and out-of-plane equilibrium of the web and air lubrication effects. The problem is non-dimensionalized and the effects of the non-dimensional parameters on traction are investigated for a range of practical values.

1. INTRODUCTION

Web traction over rollers is known to deteriorate due to air entrainment at high web speeds [1-2]. In this paper, a general model is presented to predict the traction capability of an impermeable web over a smooth roller. A web, guided by an ideal roller without any bearing resistance and operating in vacuum should not experience any slip over the roller, if the friction coefficient is not zero[3]. At high web speeds, the amount of entrained air and the magnitude of air pressure in the web-roller interface increases. This results in, some portion of the belt-wrap pressure, T/R_r, to be shared by the rigid body contact pressure. Thus, by considering Coulomb's friction law, $F_f = m F_n$, the frictional force required to sustain high traction can be effectively reduced, without altering the value of the coefficient of static friction, *m*.

2. WEB-ROLLER INTERFACE MODEL

The equations of equilibrium for a web travelling over a cylindrical surface at steady state as shown in Fig. 1 are given by the following coupled set of eqns. [6],[7],

$$D\frac{d^4\tilde{w}}{dx^4} + D_s\tilde{w} + \left(rcv^2 - N\right)\frac{d^2w}{dx^2} - \frac{dN}{dx}\frac{dw}{dx} = \left(p - p_a\right) + p_c - \frac{N}{R(x)} \qquad (1)$$

$$\frac{dN}{dx} + t = 0 \qquad (2)$$

where $\tilde{w} = w - w_r$ is the web deflection, with respect to

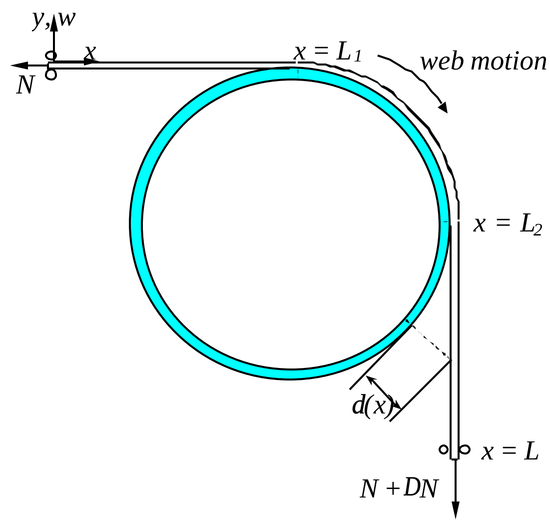

Fig. 1 Schematics of a web moving over a roller.

the equilibrium spacing w_r of a stationary web. Coulomb's law is used for the in-plane friction stress *t* due to friction,

$$t = m_s p_c \begin{cases} -1 \text{ if } v > 0 \\ 1 \text{ if } v < 0 \end{cases} \qquad (3)$$

where m_s is the "slow-speed" friction and *v* is tape speed [2]. An empirical relation is used for contact pressure [4],

$$p_c = P_0\left(1 - \frac{h}{s_0}\right)^2. \qquad (4)$$

The web-roller clearance *h* is calculated by,

$$h(x) = w(x) + d(x) \qquad (5)$$

where *d(x)* is the initial web-roller clearance. The web has simple supports on its boundaries. Upstream tension *T* is specified as the boundary condition for eqn. (2) as,

at $x = 0$: $N = T$ (for $v > 0$). (6)

Air lubrication in the interface is modeled with the Reynolds equation with first-order slip corrections [8],

$$\frac{d}{dx}\left[\left(ph^3 + 6lh^2\right)\frac{dp}{dx}\right] = 6m\left(v + v_r\right)\frac{dph}{dx}. \qquad (7)$$

On the boundaries the air pressure is set to the ambient pressure p_a. Equations (1)–(7) are coupled, non-linear equations. Their solution is obtained numerically [5].

2.1 Non-dimensional Variables

In order to reduce the number of parameters that control the problem the following non-dimensional variables are defined,

$$\bar{w} = \frac{w}{c},\ \bar{h} = \frac{h}{s_0},\ \bar{p} = \frac{p}{p_a},\ \bar{p}_c = \frac{p_c}{p_a},\ \bar{N} = \frac{N}{T} \qquad (8)$$

where $L_w = bR_r$ is the wrap length. The length coordinate is $\bar{x} = x/L_w$. Substitution of eqn. (8) in (1) – (7) results in following eleven non-dimensional control parameters,

$$S = \frac{Ec}{T\left(1-u^2\right)},\ C = \frac{c}{L_w},\ P = \frac{p_a R_r}{T},\ b = \frac{L_w}{R_r},$$

$$V = \frac{v}{\sqrt{T/cr}},\ X_L = \frac{x_L}{L_w},\ X_R = \frac{x_R}{L_w},\ \Sigma = \frac{s_0}{c},$$

$$P_0 = \frac{p_0}{p_a},\ \Lambda = \frac{l}{s_0},\ B = \frac{6m_a v L_w}{p_a s_0^2}. \qquad (9)$$

The coefficient of friction m_s remains unchanged.

3. RESULTS

In this work, the effects of nine of the eleven non-dimensional parameters described above on the traction between a roller and a flexible web are investigated numerically. The governing equations were first solved numerically as described in [5]. The results were, then evaluated by using curve fitting.

3.1 Effects of sliding on the steady state traction

Fig.2 represents the steady state conditions at the tape-roller interface for a typical mixed lubrication case. The following non-dimensional parameters were used: $B = 113.5$, $C = 1.061\times10^{-3}$, $P = 3.8$, $V = 1.94\times10^{-2}$, $b = \pi/2$, $S = 2\times10^{-2}$, $P_o = 10$, $X_L = X_R = 1.061$, $L = 1.27$. In particular, the variation of in-plane stress resultant (tension) $\bar{N}$, tape-roller spacing $\bar{h}$, contact pressure $\bar{p}_c$ and air pressure $\bar{p}$ are presented in the *wrap region* which spans $0 \le \bar{x} - \bar{L}_1 \le 1$.

For this case the tape-roller spacing is less than one in the wrap region, which indicates full rigid body contact; $\bar{h}$ varies from approximately 0.86 on the entry side ($\bar{x} = 0$) to 0.8 on the exit side. Thus it is seen that when the effects of asperity compression and in-plane stress resultant are considered no constant gap region exists in the interface. The in-plane tension $\bar{N}$ increases approximately 40% along the interface due to sliding friction causing a subsequent increase in the belt-wrap pressure. Increasing belt-wrap pressure applies higher compression on the asperities along the interface, resulting in the variable gap height indicated above. The same effect is also responsible for the slight linear increase in contact pressure $\bar{p}_c$ and air pressure $\bar{p}$.

Effects of the non-dimensional parameters (eqn. (9))

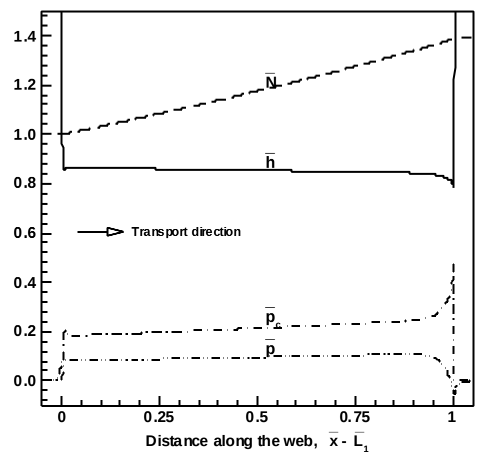

Fig. 2 Non-dim. tension $\bar{N}$, tape-roller spacing $\bar{h}$, contact $\bar{p}_c$ and air pressures $\bar{p}$ in the *wrap region*.

on the traction between the web and the roller are investigated numerically. In [9] the results are presented in graphs and in the forms of two variable polynomial curve fits. It was shown that dependence of traction on the non-dimensional transport velocity V and stiffness parameter S are weak; but traction depends strongly on the other eight parameters as shown in equations.

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