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STABILITY OF AN AXIALLY ACCELERATING STRING SUBJECTED TO FRICTIONAL GUIDING-FORCES

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ABSTRACT

The dynamic response of an axially translating continuum subjected to the combined effects of a pair of spring supported frictional guides and axial acceleration is investigated; such systems are both non-conservative and gyroscopic. The continuum is modeled as a tensioned string translating between two rigid supports with a time dependent velocity profile. The equations of motion are derived with the extended Hamilton's principle and discretized in the space domain with the finite element method. The stability of the system is analyzed with the Floquet theory for cases where the transport velocity is a periodic function of time. Direct time integration using an adaptive step Runge-Kutta algorithm is used to verify the results of the Floquet theory. Results are given in the form of time history diagrams and instability point grids for different sets of parameters such as the location of the stationary load, the stiffness of the elastic support, and the values of initial tension. This work showed that presence of friction adversely affects stability, but using non-zero spring stiffness on the guiding force has a stabilizing effect.

INTRODUCTION

Dynamics of translating continua has been investigated in the past forty years because of the large number of applications that are encountered in mechanical systems such as power transmission chains and belts, band saw blades, textile fibers, magnetic tapes, paper sheets, thread lines, elevator cables and pipes conveying fluids [1]. Excessive vibrations are usually to be avoided in axially travelling structures; in magnetic tape drives, for example, they cause imperfections on the magnetic signal,while in band saws they result in poor cutting quality.

Miranker was the first to derive the equation of motion for an axially accelerating string [2]. An approximate solution for an accelerating string driven harmonically at one end was later given by Mote [3]. More recently Pakdemirli et al. applied the Floquet theory to analyze the stability of a string moving with a prescribed sinusoidal velocity function [4]; Pakdemirli and Batan analyzed stability for the case with periodic constant acceleration-deceleration profile [5]. Pakdemirli and Ulsoy applied the method of multiple scales when the axial velocity of

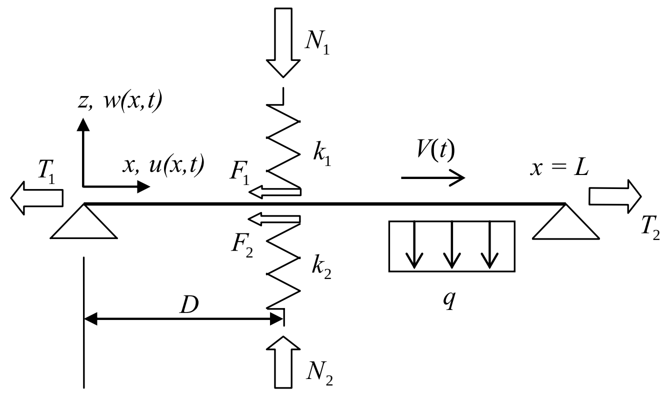

Fig. 1 Schematic description of the traveling string.

the string is assumed to have small harmonic variations about a constant mean velocity [6].

No friction is involved in the studies mentioned above. Cheng and Perkins derived exact solutions through separation of variable for an axially moving string subject to a dry friction guide, in case of constant transport velocity [7]. Zen and Müftü investigated this problem using the finite element method and α-method of time integration [8].

FORMULATION

For a continuum moving between two pulleys, one of which is supported by a spring with stiffness k_s, the equation of motion is given by the following equations:

$$\rho\left(w_{,tt}+2Vw_{,xt}+V^2w_{,xx}+V_{,t}w_{,x}\right)-\left(T_1+\left[F_1+F_2+\left(k_1-k_2\right)w_D\right]H\left(x-D\right)\right)w_{,xx}+\left(k_1+k_2\right)w\delta\left(x-D\right)=0 \quad (1)$$

$$T_1 = T_0+\eta\rho V^2 \quad (2)$$

where ρ is the is the mass of the string per unit length, $V(t)$ = $V_0\sin(\omega_0 t)$, is the time dependent string transport velocity, μ is the dynamic friction coefficient, δ is the Dirac delta function, H Heaviside step function, T_0 is the initial string tension, η = $[1+k_sL/EA]^{-1}$ and the other parameters are defined on Fig. 1. The stability of the system is analyzed with the Floquet theory for cases where the transport velocity is a periodic function of time. Direct time integration using an adaptive step Runge-Kutta algorithm is used to verify the results of the Floquet theory. Results are given in the form of time history diagrams and instability point grids for different sets of parameters such as the location of the stationary load, the stiffness of the elastic support, and the values of initial tension [9].

RESULTS

The effects of friction and guiding forces on the stability of a tensioned, traveling string is investigated by Floquet analysis [9]. The analysis is carried-out for the transport speed

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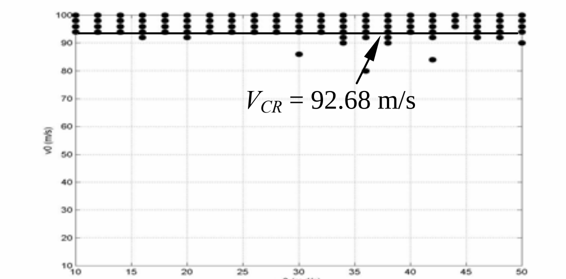
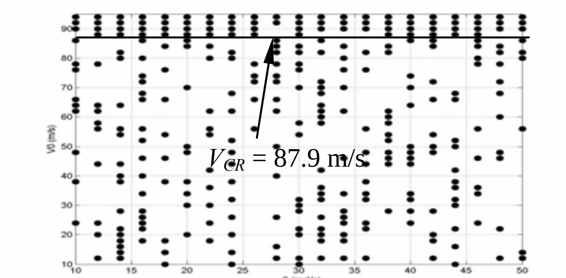
magnitudes that are in the range $10 \le V_0 \le 100$ m/s and for the speed fluctuation frequencies in the range $10 \le \omega_0 \le 100$ rad/s. The evaluation is made for the base parameter given in Table 1. The results are presented in $\omega_0 - V_0$ plots in Fig. 2, where black circles correspond to unstable data points. These plots show that, when the frictional guide is introduced (Figs. 2b,c), the system becomes unstable at many more points as compared to the non-guided system (Fig. 2a). The frictional instabilities are weaker than the buckling instabilities which occur when the string is translating faster than the critical speed $V_{cr} = (T_0 / \rho(1-\eta))^{1/2}$ [9]. Nevertheless, the number of unstable points increase with increasing μ.

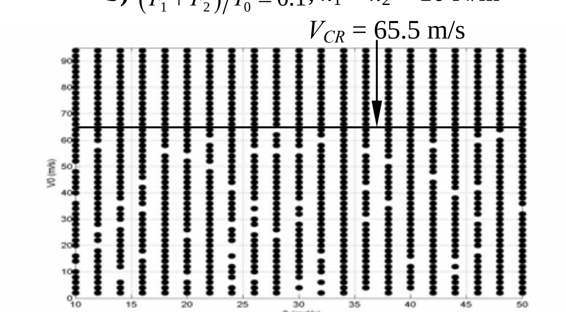
An interesting observation in Fig 2a, for the non-guided string, is the presence of unstable points even for $V < V_{cr}$ and again presence of stable points for $V < V_{cr}$. This effect is due to the transient nature of the translation speed. The transient displacement of the middle point of the string is plotted in Fig. 3 for two of these points and confirm the predictions of the Floquet theory.



a) $\mu = 0$, $k_1 = k_2 = 0$



b) $(F_1 + F_2)/T_0 = 0.1$, $k_1 = k_2 = 10$ N/m



c) $(F_1 + F_2)/T_0 = 0.5$, $k_1 = k_2 = 10$ N/m

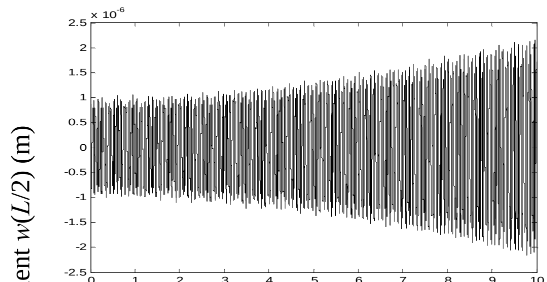
Fig. 2. Effect of friction on stability of the accelerating string. Black dots indicate unstable combinations.

L (m)	T_0 (N)	ρ (kg/m)	η
0.3681	76.22	4.032e-2	0.78

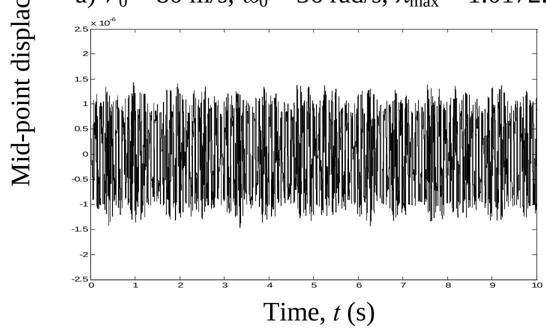
Table 1. Base parameters used in the paper.

REFERENCES

1. Wickert, J.A., and Mote Jr., C.D., 1988, "Current Research onthe Vibration and Stability of Axially Moving Materials," *Shock and Vibration Digest* 20-5 pp. 3-13.
2. W.L. Miranker, 1960, "The Wave Equation in a Medium in Motion," *IBM Journal of Res and Dev* 4-1, pp. 36-42.
3. Mote Jr., C.D., 1975, "Stability of Systems transporting accelerating axially moving materials," *Journal of Dynamic Systems, Measurements and Control, Acoustics,* 97pp. 96-98.
4. M. Pakdemirli, A.G. Ulsoy, A. Ceranoglu, 1994, "Transverse vibration of an axially accelerating string," *Journal of Sound and Vibration* 169-2, pp. 179-196.
5. M. Pakdemirli, H. Batan, 1993, "Dynamic Stability of a Constantly Accelerating Strip." *Journal of Sound and Vibration* 168-2, pp. 371-378.
6. M. Pakdemirli, G.Ulsoy, 1977, "Stability Analysis of an Axially Accelerating String," *Journal of Sound and Vibration* 203-5, pp. 815-832.
7. S.P. Cheng, N.C. Perkins, 1991, "The Vibration and Stability of a Friction-Guided, Translating String," *Journal of Sound and Vibration* 144, pp 281-292.
8. G. Zen, S. Müftü, 2003, "Friction Induced Transverse Vibrations of an Axially Accelerating String," *Proc 2003 STLE/ASME Int. Joint Trib. Con.* CD-ROM 2003TRIB-262.
9. G. Zen, S. Müftü, 2005, "Stability of an Axially Accelerating String Subjected to Frictional Guiding-Forces," *Journal of Sound and Vibration*, accepted for publication.



a) $V_0 = 80$ m/s, $\omega_0 = 36$ rad/s, $\lambda_{max} = 1.0172$.



b) $V_0 = 94$ m/s, $\omega_0 = 44$ rad/s, $\lambda_{max} = 1.0000$.

Fig. 3 Transient response of two ω_0-V_0 combinations from Fig 2a.

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