

An Introductory Overview of Contact Mechanics and Adhesion

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Abstract

Adhesion plays a large role in contact mechanics. Classical Hertzian contact theory does not take adhesion into consideration. Instead it just looks at the mechanical aspects of the deformation. Over the years researchers have realized that surface forces play a much larger role than originally expected. All of the current models use the Hertzian model as the basis of their models. There are currently 3 main models used by tribologists, all of which take a different approach to solving the problem. The DMT (Derjagin, Muller, Toropov) and JKR (Johnson, Kendall, Roberts) came out at about the same time. The DMT model assumes that the indenter has a small radius of curvature and high stiffness. The JKR model assumes that the indenter has a large radius of curvature and a low stiffness. Both of these solutions have been validated and work in certain situations. The problem is there is a grey area between the two models. The newest, and considered the most accurate, model was created by Maugis. The Maugis model works in a much larger range of material properties and can properly describe both the JKR and DMT model.

1.0 INTRODUCTION

Contact mechanics covers a wide range of mechanical interactions. At the macro scale, in contact mechanics, it is assumed that the surfaces are smooth and in complete contact. When a problem is looked at in this scale it is assumed that the entire surface will fail, or plastically deform, as a whole. If the focus changes to the micro, and eventually the nano scale, tribologists start to look at contact in a different manner. In the micro and nano scale they no longer look at the surfaces as smooth flat surface. When you get down to the micro scale the surface starts to look more and more like a mountain range. At this scale these peaks and valleys play a big role in contact. On this scale a surface can start to fail, or enter the plastic region, long before the part fails as a whole. To properly understand the interactions of these peaks, or asperities, with one and other one has to take into account more than just the mechanics of the interaction. At this scale, micro and nano, adhesion plays a large role in the interactions of the surfaces.

2.0 ADHESION MODELS

The shortest range of interaction is governed by molecular forces. These forces induce strong attraction in the spacing between the solids however the spacing between the surfaces must be less than 10nm. Attractive interactions between electrically neutral particles can occur in diverse ways and this type of attraction is called the van der Waals interaction.

2.1 Van der Waals

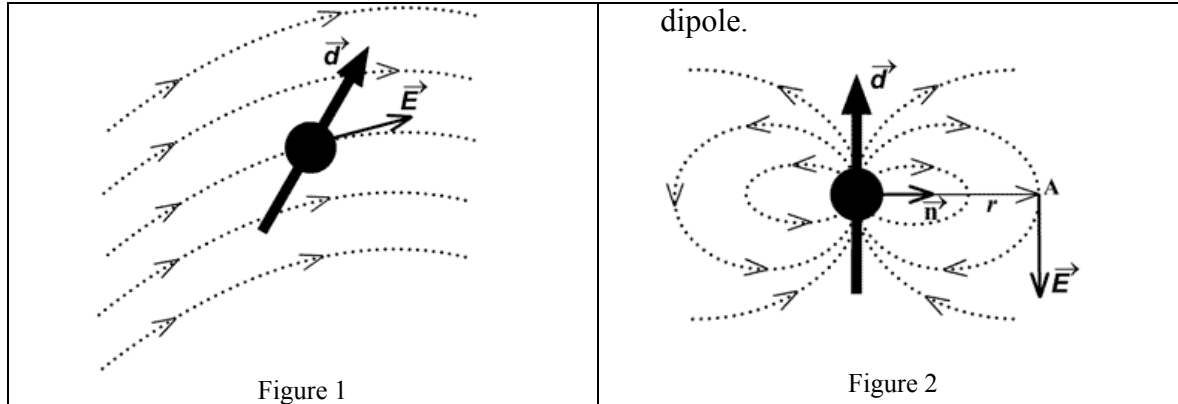
The Van der Waals force or the intermolecular attractive force has three components. All of which have slightly different physical nature but have the same potential dependence on the intermolecular distance $\left(\frac{1}{r^6}\right)$, where r is the distance between the interacting surfaces. This dependence on the intermolecular distance allows the direct comparison of the constants of interaction that correspond to three Van der Waals force components. This is because of the proportions between the components will be held constant at different magnitudes, d . Constants at $\left(\frac{1}{r^6}\right)$ multiplier will differ for various materials.

$$W = W_{orient} + W_{ind} + W_{disp} \propto 1/r^6 \quad (1)$$

Where W_{orient} , W_{ind} and W_{disp} are the orientation dependant interactions between the surfaces (also known as the Casimir force). The induction interaction (Debye force) and The dispersion interaction (London force) respectively .

In order to properly introduce the three Van der Waals force components, which are based on dipole interaction, therefore the two basic formulas should be better understood.

<p>1) the energy of dipole \mathbf{d} placed in field \mathbf{E}</p> $W_D = -\mathbf{d} \cdot \mathbf{E} \quad (2)$	<p>2) the electric field produced by the dipole \mathbf{d} is</p> $\mathbf{E} = \frac{3(\mathbf{nd})\mathbf{n} - \mathbf{d}}{r^3}, \quad \mathbf{E} \propto \frac{1}{r^3} \quad (3)$ <p>(n) Unit vector directed from the point at which the energy is determined to the</p>
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The **orientational interaction** (Casimir force) arises between two polar molecules, each of which has the electric dipole moment. In accordance with equations (2), (3) the interaction energy of dipoles d_1 and d_2 separated by distance r

$$W_D = \frac{d_1 d_2 - 3(d_1 n)(d_2 n)}{r^3} \propto \frac{1}{r^3} \quad (4)$$

This energy depends sufficiently upon the molecules relative position. Here \mathbf{n} is the unit vector directed along the line between molecules.

In order to reach the potential minimum, dipoles tend to align along the common axis. The thermal motion, however, breaks this order. To determine the resulting orientation potential W_{orient} should average statistically interactions with the use of the Gibbs distribution over all possible orientations of molecules pair. The energy of the dipoles, shown in Figure 1, depends on their mutual orientation. To find the effective potential it is necessary to perform the thermodynamic averaging overall the directions of the dipoles in space.

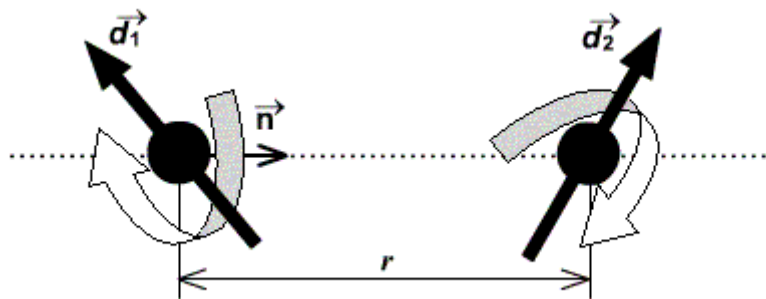


Figure 3. The energy of dipoles interaction

Introducing constant A_1 in accordance with (4), we can finally have:

$$W_{orient} = \frac{-A_1}{r^6} \quad (5)$$

The **induction interaction** (Debye force) arises between polar and non-polar molecules. The Electric field, E , generated by dipole d_1 polarizes the other molecule, shown in Figure 4. Under the polar molecule field the neighboring one acquires the induced dipole moment. The induced moment calculated in the first order of the quantum perturbation theory is equal to $d_{ind} = \chi E$ where χ stands for the molecules ability to polarize.

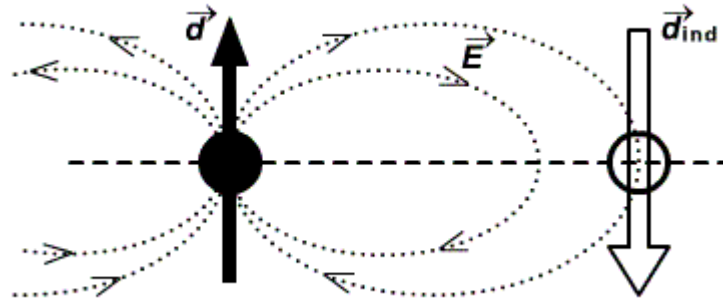


Figure 4. Induced Dipole Moments

Then, the potential of induction interaction is computed as

$$W_{ind} = d_{ind} E \propto \frac{1}{r^6} \quad (6)$$

The **dispersion interaction** (London force) is the prevailing force because it involves the non-polar molecules as well. This third term in (1) is always present and is considered the driving force behind the Van der Waals force. .

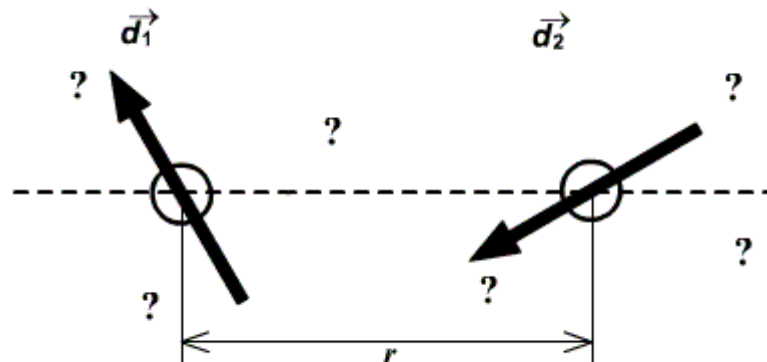


Figure 5. Non-polar Dipole Interactions

Due to the quantum uncertainty the non-polar molecules have a momentary dipole moments and interactions By using quantum perturbation theory we can get

$$W_{disp} \propto W_D^2 \quad \text{and} \quad W_{disp} = \frac{-A_3}{r^6} \quad (7)$$

Where the constant A_3 is called the Hamaker constant. The Hamaker constant is considered to be almost as important, if not more, than the surface energies of the materials. The dipole

moment of one molecule arises from the fluctuations that generates the field which, in turn, polarizes the second molecule. The already nonzero field of the second molecule polarizes the first one.

The potential of the molecules pair wise interaction depends on the distance as r^6 . The corresponding force is equal to its derivative with respect to distance r as

$$f = \frac{6A_3}{r^7} \quad (8)$$

and it is shown that The attraction force (8) decays sharply with distance (r^{-7}). Basing on this microscopic description we can determine the attraction force between the indenter and the flat surface by

$$F = \sum_{\substack{\text{indenter} \\ \text{molecules}}} \left(\sum_{\substack{\text{flat surface} \\ \text{molecules}}} f \right) \quad (9)$$

It is clear that this result will depend sufficiently on the problem geometry.

Van der Waals forces consist of a repulsive and an attractive contribution. The repulsive part is due to the effect that two molecules cannot penetrate each other, while the attractive part occurs between neutral molecules, which don't carry a fixed dipole moment. Quantum fluctuations induce transient dipole moments, which lead to mutual attraction.

2.2 The nature of adhesion

There are two main cases to look at when describing the effects of the Van der Waals forces on an indenter interacting with a flat surface. The first is the long range Van der Waals forces that are acting on the indenter before contact has been made. The second is the elastic forces acting between the surfaces after contact has been made. There is also a middle range where attraction forces between the indenter and the flat surface molecule pairs act (potential is proportional to $\frac{-1}{r^6}$) and repulsive forces between some other pairs act too (potential is proportional to $\frac{1}{r^{12}}$). Because of this it is impossible to find the interaction force between the whole indenter and the flat surface.

In the transition region a qualitatively new phenomenon, adhesion, arises. It originates from the short-range molecular forces. When the indenter asperity approaches the flat surface Van der Waals forces start acting upon the indenter. These Van der Waals forces have a sufficient range and can be felt at the distance of a few tens of angstroms. Then at the distance of several angstroms repulsive force starts acting.

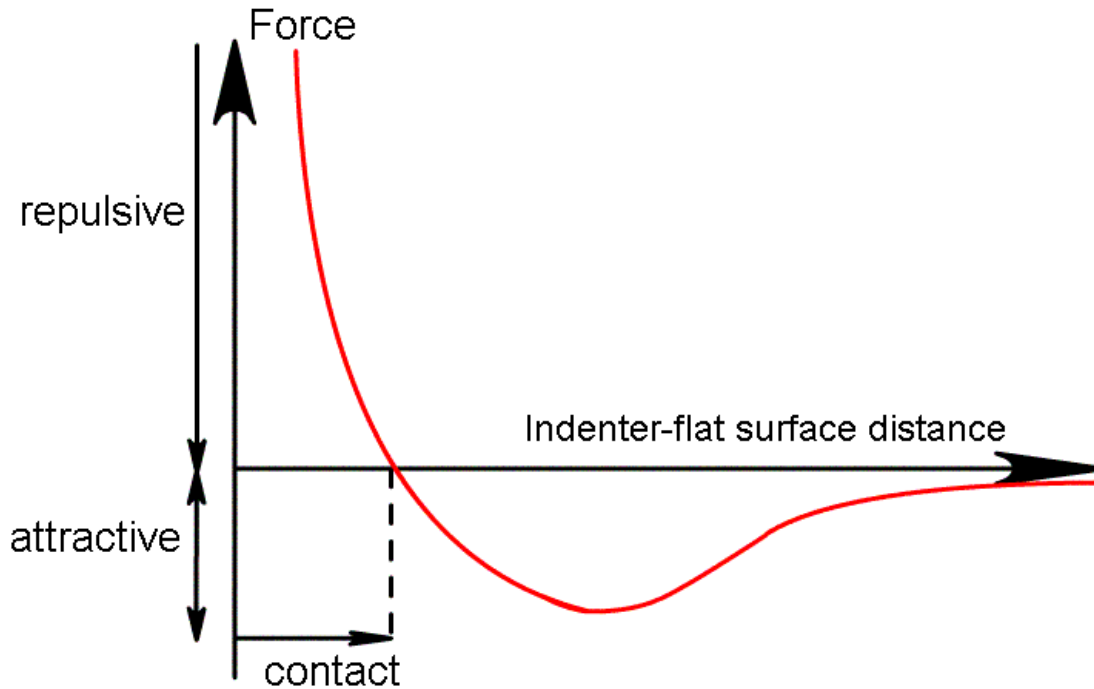


Figure 6. Interaction Forces

In real world conditions (ambient air) there is always some humidity present in the air. This means there is a monolayer of water on the surfaces. This water layer is typically on both the indenter and the flat surface. When indenter comes into contact with the monolayer of water a capillary force arises which increases the attractive force between the surfaces. This capillary force will be described in greater detail in a later section. Interaction between the indenter and the flat surface may appear rather often. This can be both attraction and repulsion.

To distinguish between these two types of adhesion, indenter-liquid film interaction on the surface and the indenter - flat surface interactions. Assuming the first case is a capillary force interaction, the adhesion forces between the indenter and the flat surface are caused by the molecular electrostatic interaction.

Adhesion is a non-conservative process. Forces acting during the interaction between the indenter and the flat surface differs from the forces during the indenter asperity retraction, shown in figure 7. Such an operation requires some work to be done which is called the work of adhesion. The work of adhesion and its dependences on the forces acting on the indenter and the flat surface distance during approach-retract cycle are shown.

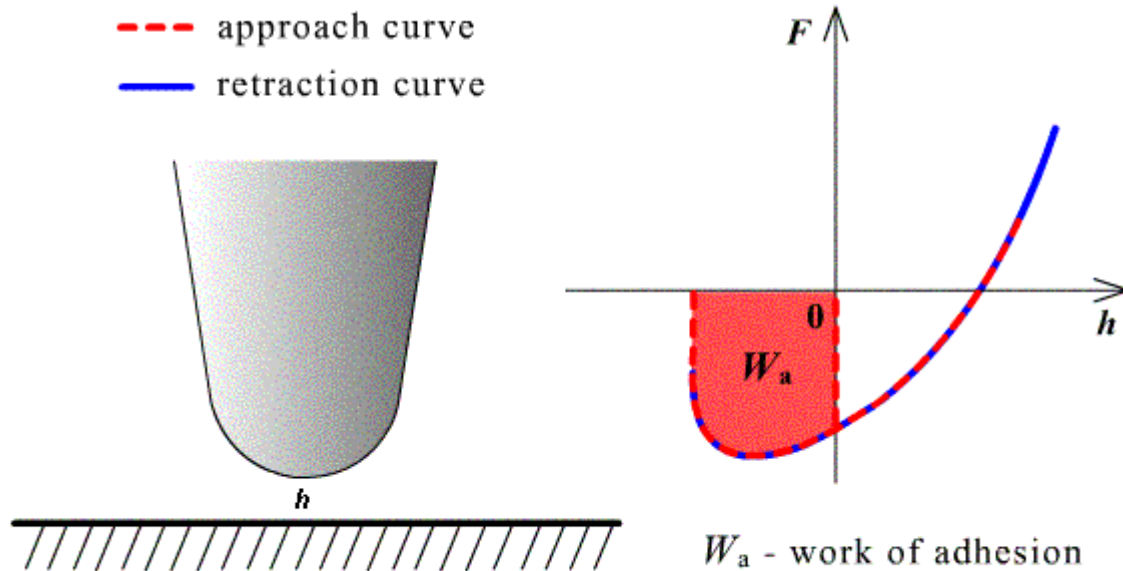


Figure 7. Work of Adhesion Curves

Adhesion is the irreversible process and the forces are different during the approach for those felt during the retraction. To describe the adhesion quantitatively there are some available models for approximation. For solids there are various corrections to the Hertz solution for finding the contact area in the presents of Van der Waals attraction forces, capillary, molecular electrostatic interaction and repulsion forces. This is of coarse referring to the point where the indenter touches the flat surface and external force acting upon the indenter and the surface are in equilibrium.

Adhesion is sticking of two surfaces in contact due to electrostatic forces having different nature for different materials. Adhesion is a non-conservative process, therefore, to separate surfaces one needs to expend an additional work. In the contact zone a "neck" arises.

2.3 Capillary Force

One of the forces commonly evaluated with adhesion is known as the capillary force. . Capillary forces are caused by the presense of liquids interacting between the contacting surfaces. As mentioned before in a real world environment humidity is always present. For this model we will assume we have a spherical indenter that has been dipped into a thin film of water on a flat surface. Assuming the surface is hydrophilic, or the interface contact angle is less than 90° , when we pull the indenter out of the film a neck will form. This neck is what actually causes the capillary force. This can be shown in figure 8.

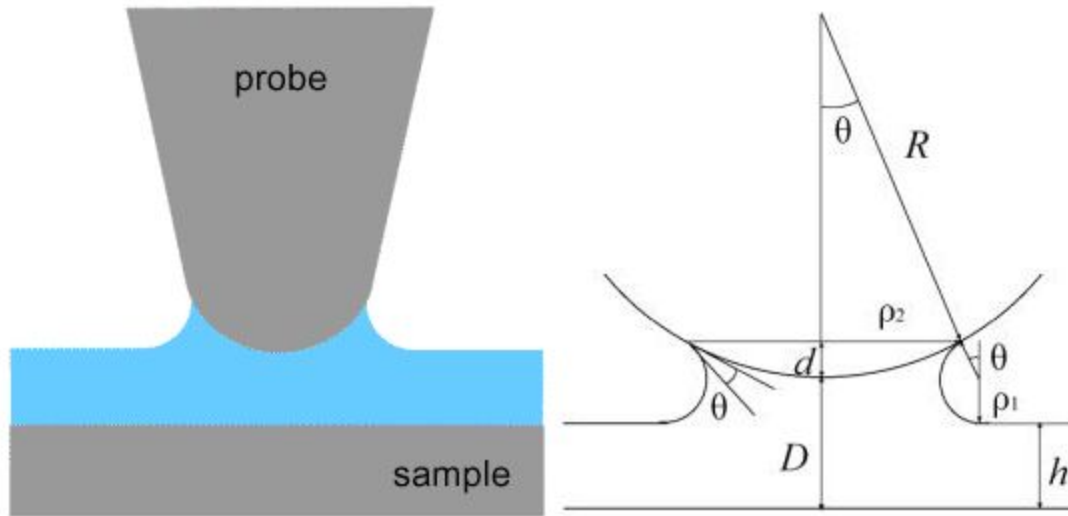


Figure 8. Capillary Forces

To properly understand the effect of capillary forces a basic understanding of the geometry must be formulated. This can be seen in figure 8. Let the radius of the spherical indenter be much larger than any of the other characteristic dimensions. The capillary force is a function of the separation distance between the surface and the indenter (D), the immersion depth (d), film thickness (h), liquid surface radius (ρ_1) and radius of the spherical indenter (ρ_2) and the surface tension of the liquid (σ). After some assumptions are made and some basic algebra we can come up with the following:

$$F_{cap} = 4\pi R\sigma(1 + \cos\theta) \quad (10)$$

3.0 CONTACT MECHANICS

When two bodies come into contact with one another the initial contact starts with a point or a line of contact. Increasing the force on the bodies, in the form of compression, will result in an increase of the contact area and cause the deformation within the bodies. The continuum models can describe the mechanical contact between solid bodies. The main models all neglect the atomic details.

3.1 The Hertz Model

The theory of Hertz contact considers the interaction between two spheres. This theory is intended to be used with dry surfaces and no adhesion. Others have come up with different models that incorporate different types of adhesion. The effective radius (11)

$$\frac{1}{R} = \frac{1}{R_A} + \frac{1}{R_B} \quad (11)$$

and the effective elastic modulus (12)

$$\frac{1}{K} = \frac{3}{4} \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right) \quad (12)$$

are functions of the different surfaces where E is the Young's modulus and ν is the Poisson's ratio. This applied load will cause a finite contact area. This can be seen in figure 9.

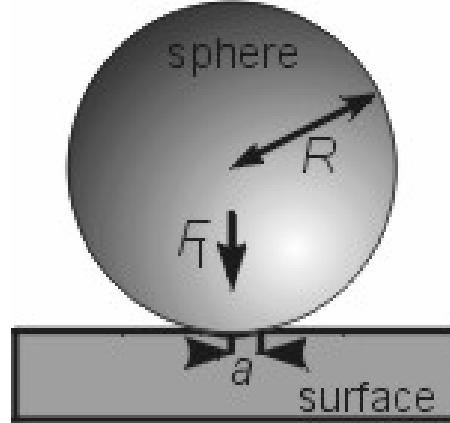


Figure 9. Basic Hertz Contact Model

In this case consider the contact between a sphere and a plate, instead of two spheres, equation (11) becomes $R = r_1$. Hertz found the applied force had a direct relationship to the contact radius

$$F = \frac{K \cdot a^3}{R} \quad (13)$$

a function of the effective radius, applied force (F_n) and effective modulus. The indentation depth (14)

$$h = \frac{a^2}{R} \quad (14)$$

is a function of contact area and the radius of curvature. The contact radius (a) can be found in equation (15).

$$A_c = \pi a^2 \Rightarrow a^2 = \left(\frac{R \cdot F}{K} \right)^{2/3} \quad (15)$$

Using these equations Hertz proved that the contact area was directly dependent on the applied force. These equations also show that if the force is removed, while still in the plastic zone and assuming there are no hysteretic effects, the ball will return to its original state.

3.2 The DMT Model (Derjagin, Muller, Toropov – 1975):

This model can be applied to indenters with small curvature radius and high stiffness. It is assumed that deformed surfaces geometry is very similar the that found in by the Hertz solution. Consideration of the Van der Waals forces acting along the contact area perimeter results in an additional attractive force between the indenter and the surface. This attractive force weakens the forces of elastic repulsion. This relationship and the relationship between the applied force and penetration depth can be seen in figure

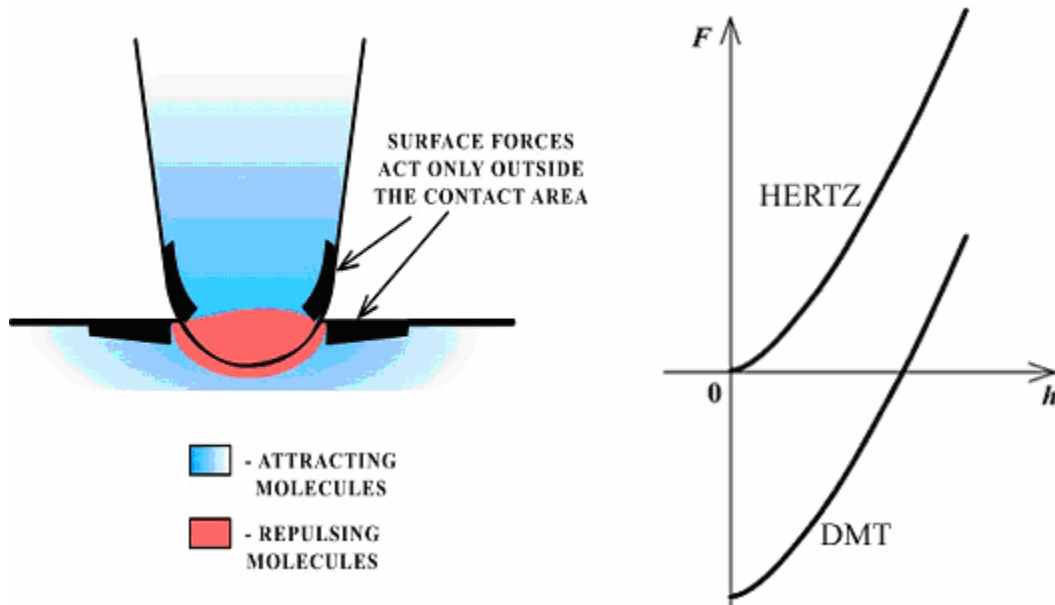


Figure 10. Surface Forces and Indenter Depth-Force Relationship

The relation between the force and the penetration depth is as follows:

$$F = \frac{K a^3}{R} - 2\pi R W_a \quad (16)$$

Where the first term is similar to Hertz solution and the second comes from the addition of adhesion; K is the effective elastic modulus, a is the radius of the contact area, R is the radius of the indenter asperity and the penetration depth, which is the same as the Hertz solution in this case, is given by

$$h = \frac{a^2}{R} \quad (17)$$

3.3 The JKR model (Johnson, Kendall, Roberts – 1964-1971)

This model is applied when the indenter has a large curvature radius and low stiffness. Such systems are called strongly adhesive. The model accounts for the influence of Van der Waals forces within the contact zone. This and the force vs penetration depth can be found in figure 11.

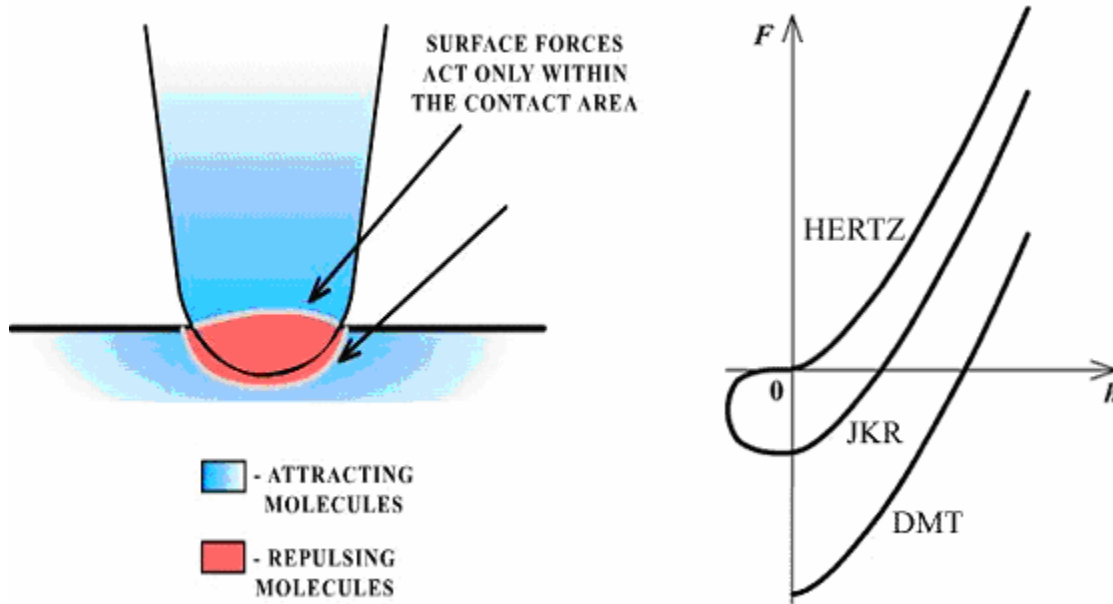


Figure 11 Surface Forces and Force vs Indenter Depth

Due to the Van der Waals force the attraction between the indenter and the surface rises. This increase in attractive force not only weakens the force of elastic repulsion (shown in figure 11) but results in the neck creation (also shown in figure 11) in the repulsive force region.

The relation between the force and the penetration depth is as follows:

$$F = \frac{K a^3}{R} - \sqrt{6 \pi W_a a^3} \quad (18)$$

Where the first term is similar to Hertz solution and the second comes from the adhesion. In this case the and the penetration depth is different than that found by Hertz and it is given by

$$h = \frac{a^2}{R} - \frac{2}{3} \sqrt{\frac{6 \pi W_a a}{K}} \quad (19)$$

3.4 The Maugis Model (1992)

This is the most universal and accurate approach at this point in time. It can be applied to any system (any materials) and works with a wide range of adhesion. The amount of adhesion in the system can be determined by the parameter λ , where ζ_0 is the inter-atomic distance

$$\lambda = \frac{2.06}{\zeta_0} \left(\frac{R W_a^2}{\pi K^2} \right)^{1/3} \quad (20)$$

DMT and JKR models are extreme cases of the Maugis mechanics corresponding to different parameters λ . For the stiff materials (DMT) $\lambda \rightarrow 0$, for compliant materials (JKR) $\lambda \rightarrow \infty$.

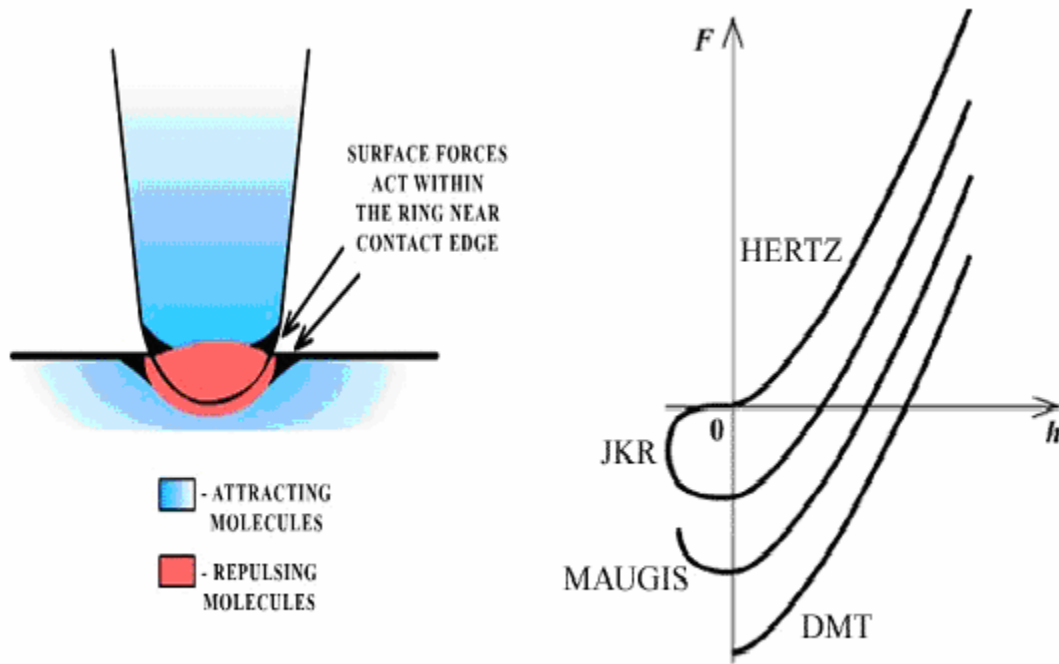


Figure 12. Surface Forces and Force vs Indenter Depth

The model forces within the contact zone and the force vs. indentation depth can be found in figure 12.

The Maugis model assumes that the molecular attraction force acts within a ring zone at the contact area border. The Maugis correction to the Hertz problem solution is expressed implicitly via parameter m :

$$1 = \frac{\lambda a^2}{2} \left(\frac{K}{\pi R^2 W_a} \right)^{\frac{2}{3}} \left[\sqrt{m^2 - 1} + (m^2 - 2) \arctan \sqrt{m^2 - 1} \right] + \frac{4\lambda a^2}{3} \left(\frac{K}{\pi R^2 W_a} \right)^{\frac{1}{3}} \left[1 - m + \sqrt{m^2 - 1} \arctan \sqrt{m^2 - 1} \right] \quad (21)$$

Where the force is given by

$$F = \frac{K a^3}{R} + \lambda a^2 \left(\frac{\pi W_a K^2}{R} \right)^{\frac{2}{3}} \left[\sqrt{m^2 - 1} + m^2 \arctan \sqrt{m^2 - 1} \right] \quad (22)$$

and the penetration depth as

$$h = \frac{a^2}{R} - \frac{4\lambda a}{3} \left(\frac{\pi W_a}{R K} \right)^{\frac{1}{3}} \sqrt{m^2 - 1} \quad (23)$$

Both JKR model and Maugis model adopt originally the existence of hysteresis during the approach-retraction cycle. This cycle can be seen in figure 13. It is assumed that during the indenters approach the attraction force rises sharply up to the moment where they contact. Then the system proceeds from point 0 into point 1. During the indenter asperity retraction the system describes the other path 1-2 until the jump out of the contact occurs 2-3. The loop 0-1-2-3 in the plot means that to separate the indenter asperity from the flat surface some work must be done which is equal to the loop area. The area within the loop is the work of adhesion .

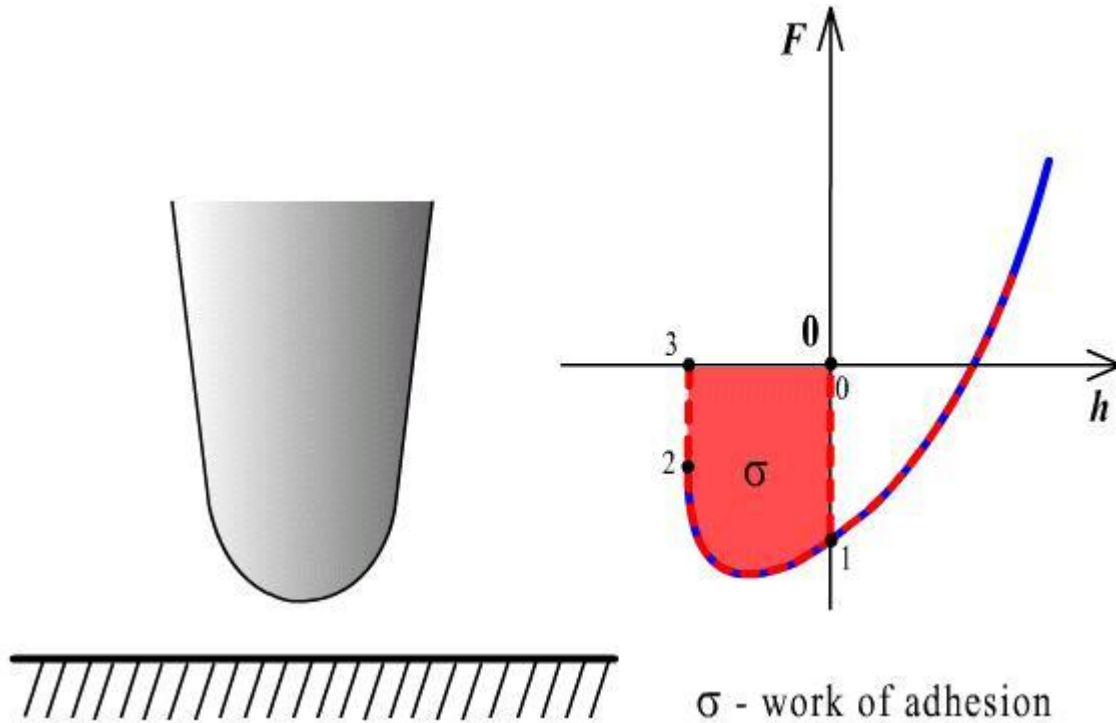


Figure 13. Indenter - Surface Reaction Force Loop

4.0 COMPARISON OF DMT, JKR AND MAUGIS MODELS

To compare the foregoing models we introduce the normalized radius of the contact area \bar{A} , force \bar{F} and penetration depth \bar{h} as

$$\bar{A} = \frac{a}{\left(\frac{\pi R^2 W_a}{K}\right)^{\frac{1}{3}}} \quad (24)$$

$$\bar{F} = \frac{F}{\pi R W_a} \quad (25)$$

$$\bar{h} = \frac{h}{\left(\frac{\pi^2 R W_a^2}{K^2}\right)^{\frac{1}{3}}} \quad (26)$$

An overall comparison of the different models, including the assumptions and restrictions, or limitations, can be found in table 1.

Table 1. Model Assumptions and Restrictions

Model	Assumptions	Restrictions
Hertz	No surface forces	Not applied to small loads in the presence of surface forces
DMT	Long-range surface forces act only outside the contact area. Model geometry is as in the Hertz model	Contact area can be decreased due to the limited geometry. Applied only to small λ
JKR	Short-range surface forces act only within the contact area	Force magnitude can be decreased due to surface forces. Applied only to large λ
Maugis	Asperity-flat surface interface is modeled as a ring.	The solution is analytical but equations are parametric. Applied to all λ values.

Table 2 shows a comparison of the normalized equations for each of the different models.

Table 2. Normalized Equations

Model	Normalized equations of the quantitative adhesion models
Hertz	$\bar{F} = \bar{A}^3$ $\bar{h} = \bar{A}^2$
DMT	$\bar{F} = \bar{A}^3 - 2$ $\bar{h} = \bar{A}^2$
JKR	$\bar{F} = \bar{A}^3 - \bar{A}\sqrt{6\bar{A}}$ $\bar{h} = \bar{A}^2 - \frac{2\sqrt{6\bar{A}}}{3}$
Maugis	$1 = \frac{\lambda\bar{A}^2}{2} \left[\sqrt{m^2 - 1} + (m^2 - 2) \arctan \sqrt{m^2 - 1} \right]$ $+ \frac{4\lambda\bar{A}^2}{3} \left[1 - m + \sqrt{m^2 - 1} \arctan \sqrt{m^2 - 1} \right]$ $\bar{F} = \bar{A}^3 - \lambda\bar{A}^2 \left[\sqrt{m^2 - 1} + m^2 \arctan \sqrt{m^2 - 1} \right]$ $\bar{h} = \bar{A}^2 - \frac{4\lambda\bar{A}}{3} \sqrt{m^2 - 1}$

As shown in FigureThe following figure shows plots of normalized force vs. normalized penetration depth for DMT, JKR and Maugis models at different λ . As can be seen, at small λ the Maugis model approaches the DMT model while at large λ approach the JKR model. This is indicative of how powerful and useful the Maugis model is. Then now several theoretical models of adhesion having different ranges of application are proposed. The most accurate one is the Maugis model.

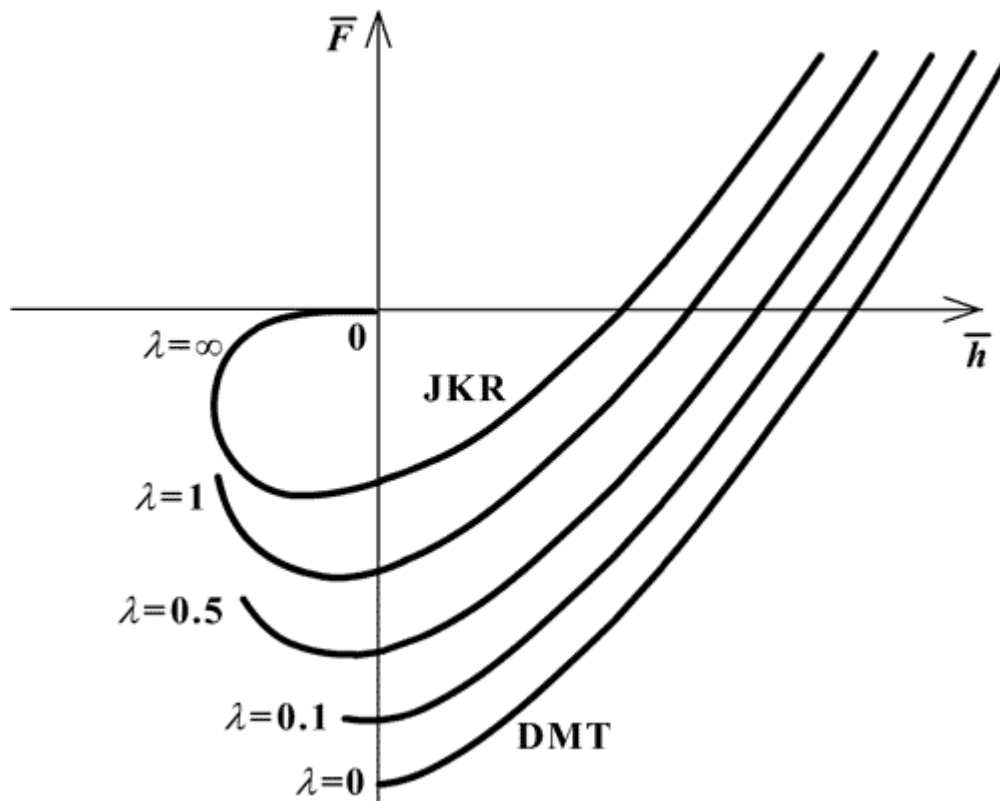


Figure 14. Comparison of the Different Contact Models Involving Adhesion

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